AN ANALYTICAL INVESTIGATION OF DYNAMIC LOADS ON INVOLUTE SPUR GEARS

By
HANAMANTA G. HUNASHIKATTI

DEPARTMENT OF MECHANICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY KANPUR

JULY, 1973

AN ANALYTICAL INVESTIGATION OF DYNAMIC LOADS ON INVOLUTE SPUR GEARS

A Thesis Submitted
In Partial Fulfilment of the Requirements
for the Degree of
MASTER OF TECHNOLOGY

By
HANAMANTA G. HUNASHIKATTI

to the

DEPARTMENT OF MECHANICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY KANPUR

JULY, 1973

TO MY PARENTS

CERTIFICATE

Certified that this work on "An Analytical Investigation of the Dynamic Loads on Involute Spur Gears" by Hanumanta. G. Hunashikatti has been carried out under my supervision and that this has not been submitted elsewhere for a degree.

DR. J. CHAKRABORTY
Assistant Professor
Dept. of Mechanical Engineering
Indian Institute of Technology
Kanpur

ACKNOWLEDGEMENT

I would like to express my deep appreciation and gratitude to Dr. J. Chakraborty for his able guidance and constant encouragement throughout this investigation.

My thanks are due to Mr. B.S. Bhadoria, Mr. S.G. Dhande and Mr. A.K. Khare for the useful and lively discussions, we used to have, on the subject.

My thanks are also due to my friends, who directly or indirectly helped me in this work.

Lastly, I thank Mr. Uma Raman Pandey for typing the manuscript neatly and patiently.

HANUMANTA. G. HUNASHIKATTI

CONTENTS

LIST OF FIGURES

LIST OF TABLES

SYNOPSIS

In this work, the deflections of teeth of gears in engagement are investigated for different combinations of gears. The mesh-stiffness and static relative displacements are evaluated from the deflection data. An equation of motion is formulated for the gears in engagement. The equation is solved by the Fourth Order Runge Kutta Method. The dynamic loads are evaluated and the maximum loads for different speeds and different combinations of teeth are evaluated.

CHAPTER I

INTRODUCTION

1.1 INTRODUCTION

Gears are essential elements of all machines. The efficient design and operation of most machines and instruments greatly depend upon the design of gear-transmission. Gear drives have a considerable influence on the weight, size, reliability and initial and running costs of the machines. The durability of the gear-drives depends upon the accurate prediction of the load carrying capacity of the gear-tooth. Some of the limiting design factors in specifying the load-capacity are :

1. Strength : This is the load the tooth can carry without permanent deformation or fracture. Strength failure normally occurs with a fatigue fracture initiated at or near the root fillet.
2. Profile-durability : This is the load that can be carried without damaging the profile. Normally, the limiting factor is the pitting of the profile at or or below the pitch point. Occasionally, when high overloads are experienced, plastic profile deformation is encountered.
3. Scoring : Loads exceeding this rating break down the oil film and allow the contacting surfaces to weld together, abrade, and otherwise destroy themselves. This rating is particularly to be considered

for highly loaded case-carburized and casehardened gears or gears operating at high speeds, high temperatures, and with thin oils.

4. Noise : It is the result of high speeds or heavy loads, which excite vibrations. The vibrations are also due to the errors in engagement between the meshing teeth.

All these factors are functions of the tooth loads and an accurate determination of these loads becomes very important. These loads can be classified as :

1. Transmitted load (tangential load)
2. Dynamic load.

The transmitted load is the useful force transmitted from one gear to another during action. It is the function of the external loading conditions. The dynamic load is the maximum instantaneous force acting between gears during action. The dynamic loads are mainly due to the imperfections in the production methods, the material properties and imperfect positioning. These factors produce a critical error in action which in the final analysis may produce gear tooth overloads in excess of safe loads. The dynamic loads seriously affect the load-transmitting capacity of gears and are also a source for noise. The data obtained so far on the basis of theoretical and experimental investigations of dynamic loads are not sufficiently reliable for use to be made of them in calculations for determining the

optimum dimensions and weight of gear-drives . Therefore work on the dynamic loads between gear teeth is continuously extending.

1.2 DETAILED DISCUSSIONS ON DYNAMIC-LOADS

Gears are generally designed to run at uniform speeds. Any disturbance which will affect the uniformity induces vibrations and hence gives rise to the dynamic loads. These disturbances may be due to the errors in geometrical size, form and position of its several features and also due to the elasticity of teeth. In addition to this the effect of friction and damping should also be considered for accurate evaluation of dynamic loads. The errors causing the disturbance can be classified as follows.

1.2-1 GEOMETRICAL ERRORS :

Geometrical errors in gears i.e. the deviation of the tooth profile from the desired one, the relative positioning of the teeth on the blank and the dimensions of teeth are due to the cumulative effects of the following imperfections :

1. Imperfections in cutter
2. Inhomogeneity of the material of the gear-blank
3. Error in indexing while cutting or generating
4. Inaccurate clamping of the blank, cutter, etc.

For study and measurement purposes, the errors in finished gear sets may be classified as :

(A) Profile-errors : These indicate the departure of the actual from the design profile. The principal sources of tooth profile error are -

a) Inaccuracies of the generating cutter tooth-profile

b) Errors in setting of the cutter

c) Departures from uniformity in the relative motions required for generation.

The nature and magnitude of these errors obviously depend upon the type of generating process employed, the condition of the machine and care taken in setting up. However in the absence of the knowledge of the actual distribution of these errors, for analysis purpose they are generally assumed to be a periodic function. This assumption is quite logical because any type of error-distribution can conveniently be represented by fourier series. Especially while cutting gears, the sources of error for generating one tooth will be the same for generating the other teeth also. Hence considering gear as a whole, the imperfectionswill be periodic.

(B) Pitch error : Spacing errors are largely a direct reflection of the inaccuracies in the indexing mechanism of the gear-cutting machine. Pitch errors may occur in isolated form or they may follow

some sort of distribution on the consecutive teeth.

Pitch errors increase the error-in-action. They are a potential source of dynamic load and noise. The frequency and the rate of change of the pitch errors from tooth to tooth greatly affect the dynamic response of gear-drives.

1.2-2 ERROR DUE TO ELASTICITY OF THE MATERIAL :

Because of the elasticity of the gear materials, the mating teeth are deflected in action. These errors are superimposed on the geometrical errors and change the error in action.

The deflections produced under the operating loads are not constant during the engagement of a pair of gear teeth mainly because of the change in the load position and the varying flexibility of the gear tooth along the profile. Moreover when the contact ratio is more than unity, more than one pair of teeth comes into action in the meshing-range. These pairs act as elastic springs in parallel and the resultant stiffness differs from the single-pair stiffness. The result will be a periodic variation in the combined mesh stiffness. In addition the deflections under load of the tooth pairs cause an increase in the contact ratio, the increase being proportional to the amount of deflection and hence the applied load. This periodic variation in the mesh stiffness acts as a source for vibration and hence the dynamic load.

In addition to these main sources, there are external factors which affect the dynamic load :

1. Torsional deflection and beam bending of gear-shaft
2. Mounting arrangement and alignment
3. The imbalance of rotating parts
4. Temperature differential throughout gear box
5. Varying output and input characteristics.

Actually dynamic load should be determined for each and every new gear application. This way the design engineer will have a back log of information with which to compare a given dynamic-load calculation. If a deligent procedure is set up whereby the apparatus is calculated in the design stage and observed in service in the field and maintenance and failure records are actually analysed, the dynamic load effect will soon have as much meaning for the average gear-designer as does the transmitted load.

There are, of course, many well established applications where it is not necessary that dynamic load calculations be made since past experience has already established the amount of dynamic load present. There are however, several types of gear arrangements where it is quite important that very careful consideration be given to the dynamic load effects during the design stage. In these arrangements the omission of a dynamic-load evaluation would be termed poor design practice(29) .

1.3 LITERATURE SURVEY

The load carrying capacity of gears has been a subject of interest to many an investigator. The works of Marx and Cuttler [1] and Ross [2] , Buckingham [3] , and Walker [4] are the pioneering contributions in this field. They have developed emperical expressions for dynamic factor as a function of peripherial speed. An extensive study of the dynamic loads on gears was conducted by the ASME Special Research Committee on gears headed by Earl Buckingham [31]. The findings of this study had been the basis for the later-day design practice. Similar study was conducted by Tuplin [6] and Reswick [7] . The main features of these studies are that the gears are replaced by simpler representative models, simulating the impact of the engaging teeth and the transient effect of the isolated tooth-errors was considered. No consideration was given to the actual error distribution and the stiffness variations in these investigations. However the emperical relations for dynamic factors given by these investigators are still useful for predicting the dynamic loads at preliminary design stages.

Strach [8] is the first to study the periodic excitation due to the error distribution and stiffness variation, which induce vibration and cause large dynamic loads. He considered unmodified teeth and formulated an equation of motion with a constant stiffness for a pair of teeth in contact and twice its value for double pair contact. Only the forced motion was

considered in the solution. The possibility of loss of contact was not taken into account. Zeman [9] included the effects of idealized periodic profile error but neglected the variation in stiffness. Richardson [10] studied static deflections of unmodified tooth profile. He also solved an equation similar to Strauch with an addition of the damping and backlash. Harris [11] studied the transmission characteristics of the gears with modified profiles. He drew the static transmission error curves using the tooth-deformation provided by Weber [5]. These curves were used to determine the distribution of forces between the tooth pairs. Simplified error curves including the salient features of the actual curves were used in the analytical solution. He showed that the amount of damping present in the system has an important influence on the nature and properties of the oscillations set up. Resonance frequencies were predicted. R.W. Gregory [21] in association with Harris and Munro continued the experiments of Harris and confirmed most of the predictions of Harris. Attia [16] for the first time measured the actual tooth deflections in action but most of his experiments were conducted at speeds well below the resonance speed. Utagawa and Harada [18] and Neiman and Rettig [13] studied experimentally the effects of intentionally produced pitch errors. Utagawa and Harada also obtained graphical solutions without considering the damping effect. A. Hichmaru [27] has obtained solutions considering the damping effect and the tooth modifications and measured profile error.

In the present work, the tooth deformations using the complex variable technique as applied to two dimensional theory of elasticity are studied. Ustinenko [20], for studying bending stress in gear-tooth, transformed the convex boundary line of the tooth profile into a straight boundary line by using conformal mapping. C.N. Baronet [25] used a similar transform functions with lesser number of terms and using the stress functions given by Aida and Terauchi [19] and studied the deflections of a single tooth. The work has been extended here for a pair of teeth in contact. The combined deflections including Hertzian contact deformation along the path of contact are studied. These deflection data are utilized in finding the static deflection characteristics and mesh stiffness for a pair of gear in engagement. The dynamic equation of motion considering damping, the variable mesh stiffness and error in action between contacting teeth is formulated. The equation is solved numerically on a digital computer by using Fourth order Runga Kutta method.

The following may be considered the contributions of the present dissertation.

: A comprehensive analytical study has been conducted to evaluate the combined mesh stiffness of the gear pair at different speeds and different combinations of the teeth in the pinion and the gear. Previous investigations were done on the basis of either

experimentally got or assumed combined mesh stiffness.

: A theoretical study of the increase in the contact ratio of the gears in action has been done. Hitherto only one investigator Niemann studied this aspect experimentally.

CHAPTER 2

ANALYSIS AND COMPUTATION-TECHNIQUE

2.1 DEFLECTION OF GEAR TOOTH

2.1-1 DEFLECTION DUE TO BEAM ACTION :

The tooth load across the tooth face of spur gear does not vary much. Neglecting the end effects, the load can be taken to be uniformly distributed across the face and the tooth-deflections can be analysed by using the two dimensional theory of elasticity. In the foregoing analysis, the complex variable method as applied to the plane theory of elasticity is used.

In the two dimensional theory of elasticity, the equilibrium in the absence of body forces, can be represented by the following equations 29 :

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = 0 \qquad (2.1)$$

In terms of the airy-stress function 'U', the stresses can be expressed as :

$$\sigma_x = \frac{\partial^2 U}{\partial x^2} \qquad (2.2)$$

$$\sigma_y = \frac{\partial^2 U}{\partial y^2} \qquad (2.3)$$

$$\tau_{xy} = \frac{\partial^2 U}{\partial x \partial y} \qquad (2.4)$$

so that the equations of equilibrium can be expressed as,

$$\Delta (\Delta U) = 0 \tag{2.5}$$

Thus the stress function U is a biharmonic function.

Using stress-strain relations

$$\begin{aligned} \sigma_x &= \lambda\theta + 2\mu \frac{\partial u}{\partial x} \\ \sigma_y &= \lambda\theta + 2\mu \frac{\partial v}{\partial y} \\ \tau_{xy} &= (\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}) \end{aligned} \tag{2.6}$$

where

$$\theta = \sigma_x + \sigma_y$$

$$\mu = \frac{E}{2(1+\nu)}$$

$$\lambda = \frac{E}{(1+\nu)(1-2\nu)}$$

ν = Posion's ratio.

Substituting for σ_x, σ_y, and τ_{xy} in terms of stress-function and solving for u and v, we get

$$\begin{aligned} 2\mu\frac{\partial u}{\partial x} &= \frac{\partial^2 U}{\partial y^2} - \frac{\lambda}{2(\lambda+\mu)} \Delta U \\ 2\mu\frac{\partial v}{\partial y} &= \frac{\partial^2 U}{\partial x^2} - \frac{\lambda}{2(\lambda+\mu)} \Delta U \end{aligned} \tag{2.7}$$

Now we define a harmonic function P, as

$$P = \Delta U \tag{2.8}$$

and Q-as conjugate of P, so as to satisfy the Cauchy-Reimann-conditions

$$\frac{\partial P}{\partial x} = \frac{\partial Q}{\partial y}$$

$$\frac{\partial P}{\partial y} = - \frac{\partial Q}{\partial x} \tag{2.9}$$

From these functions we define a complex function f(z) such that

$$f(z) = P + i\,\theta \tag{2.10}$$

where $z = x + iy$

This function is holomorphic in the region 's' of the body.

Defining a new complex function,

$$\psi(z) = p + iq = \frac{1}{4}\int f(z)\,dz \tag{2.11}$$

We get

$$P = 4\frac{\partial p}{\partial x} = 4\frac{\partial q}{\partial y}$$

$$Q = -4\frac{\partial p}{\partial y} = 4\frac{\partial q}{\partial x} \tag{2.12}$$

Then equations (2.7) can be expressed as,

$$2\mu\frac{\partial u}{\partial x} = -\frac{\partial^2 U}{\partial x^2} + \frac{2(\lambda+2\mu)}{(\lambda+\mu)}\frac{\partial p}{\partial x}$$

$$2\mu\frac{\partial v}{\partial x} = -\frac{\partial^2 U}{\partial y^2} + \frac{2(\lambda+2\mu)}{(\lambda+\mu)}\frac{\partial q}{\partial y} \tag{2.13}$$

Integrating and putting the constants of integration equal to zero, as they represent the rigid body motion, we get

$$
\begin{aligned}
2\mu u &= -\frac{\partial U}{\partial x} + \frac{2(\lambda + 2\mu)}{(\lambda + \mu)}\, p \\
2\mu v &= -\frac{\partial U}{\partial y} + \frac{2(\lambda + 2\mu)}{(\lambda + \mu)}\, q
\end{aligned}
\qquad (2.15)
$$

Now from the definition of (z), it can be shown that,

$$(U - px - qy) = 0$$

$$\text{i. .} \quad U = px + qy + p_1 \qquad (2.16)$$

In the complex form,

$$U = \mathrm{Re}\left[\bar{z}\,\varphi(z) + \chi(z)\right] \qquad (2.17)$$

where $p_1 = \mathrm{Re}\left[\chi(z)\right]$

By using the conjugate property

$$2U = \bar{z}\,\varphi(z) + z\,\overline{\varphi}(z) + \chi(z) + \overline{\chi}(z) \qquad (2.18)$$

Taking the partial derivatives and adding

$$\frac{\partial U}{\partial x} + i\,\frac{\partial U}{\partial y} = \varphi(z) + z\,\overline{\varphi'}(z) + \overline{\psi}(z) \qquad (2.19)$$

where $\psi(z) = \dfrac{d\chi}{dz}$

Then the displacements are given by

$$
\begin{aligned}
2\mu(u + iv) &= \left(\frac{\lambda + 3\mu}{\lambda + \mu}\right)\varphi(z) - \overline{\varphi'}(z) - \overline{\psi}(z) \\
&= (3 - 4\nu)\,\varphi(z) - z\,\overline{\varphi'}(z) - \overline{\psi}(z) \qquad (2.20)
\end{aligned}
$$

In the body space is transformed to simpler region by using a transformation function

$$z = \omega(\zeta)$$

Then,

$$\varphi'(z) = \varphi(\omega(\zeta))$$
$$= \phi'(\zeta)\,\frac{d\zeta}{dz}$$
$$= \frac{\phi'(\zeta)}{\omega'(\zeta)}$$

In the transformed co-ordinates, the displacement z-plane, we can write as

$$2\mu(u + i\,v) = (3-4\nu)\,\phi(\zeta) - \frac{\omega(\zeta)}{\overline{\omega'(\zeta)}}\,\overline{\phi'(\zeta)} - \overline{\psi}(\zeta) \qquad (2.21)$$

The resultant deformation is

$$z_1 = \sqrt{u^2 + v^2} \qquad (2.22)$$

2.1-2 TRANSFORMATION FUNCTION :

The semi-infinite space 's' in z-plane having a boundary of tooth shape is transformed into the infinite space 'R' having straight boundary line in z-plane. To achieve this Ustinenko 20 and Aida 19 have assumed a transformation function similar to the one given below :

$$z = \omega(\zeta) = \zeta - \sum_{n=1}^{4} \frac{a_n}{\zeta + b_n} \qquad (2.23)$$

This function satisfies the two necessary conditions of conformal mapping, namely

$$\lim_{\zeta \to \infty} \omega(\zeta) - \zeta = 0$$
$$\omega'(\zeta) \neq 0 \qquad (2.24)$$

where $\zeta = u + i\,v$

Here u and v do not represent the displacements but they represent the coordinates of the mapped space as shown in Fig. 2.1.

2.1.3 STRESS FUNCTIONS :

When a concentrated load P is acting on the point $(0,\alpha)$ on the boundary of a semi-infinite plate (Fig. 2-1), and forms an angle β with the x-axis in z-plane then the airy-stress function U is expressed by [19]

$$U = \frac{W}{2\pi} \operatorname{Re}\left[(z-i\alpha)\log(z-i\alpha)e^{-i\beta} - (\bar{z}+i\alpha)\log(z-i\alpha)e^{i\beta}\right] \quad (2.2$$

Then from (2.18)

$$\varphi(z) = -\frac{W}{2\pi} e^{i\beta}\log(z-i\alpha)$$
$$\chi(z) = \frac{W}{2\pi}\left[e^{-i\beta}(z-i\alpha)\log(z-i\alpha) + i\alpha e^{i\beta}\log(z-i\alpha)\right] \quad (2.2$$

If $\chi'(z) = \psi_1(z)$

then we can write

$$\psi_1(z) = \frac{W}{2\pi}\left[e^{-i\beta}\log(z-i\alpha) + e^{-i\beta} - \frac{i\alpha}{z-i\alpha}e^{i\beta}\right] \quad (2.2$$

Let ρ be a point on the boundary, 's' be the length along the boundary curve, the boundary condition to be satisfied is

$$\varphi(\rho) + \omega(\rho)\frac{\overline{\varphi'(-\rho)}}{\omega(-\rho)} + \omega(-\rho) = i\int_0^s \left[W_x(s) + iW_y(s)\right] . ds \quad (2.28$$

To satisfy the condition the stress functions are taken in the form

$$\phi = \phi_0 + \phi^*$$
$$\psi = \psi_0 + \psi^* \quad (2.2$$

such that ϕ_0, ψ_0 are having singular point on the boundary ϕ^*, and ψ^* are having no singular point and $\phi^* \to 0$, $\psi^* \to 0$ as $\zeta \to 0$

Then from (2.26), and (2.27) we get

$$\phi_o = -\frac{W}{2\pi} e^{i\beta} \log(\zeta - i\alpha)$$

$$\psi_o = \frac{W}{2\pi}\left[e^{-i\beta} \log(\zeta - i\alpha) + \frac{\bar{\omega}(-i\alpha)}{\omega'(\alpha)(\zeta+\mu)}\right] \qquad (2.30)$$

By applying the boundary condition and integrating over the boundary, Γ

$$\phi(\zeta) = \phi_o(\zeta) + \frac{1}{2\pi i}\int_{\Gamma} \frac{\omega(\rho)}{\bar{\omega}'(\rho)} \frac{\bar{\phi}(-\rho)}{(\rho-\zeta)} d\rho \qquad (2.31)$$

$$\psi(\zeta) = \frac{W}{2\pi} e^{i\beta} \log(\zeta - i\alpha) + \frac{1}{2\pi i}\int_{\Gamma} \frac{\bar{\omega}(-\rho)}{\omega(\rho)} \frac{\phi'(\ )}{(\rho-\zeta)} d\rho \qquad (2.32)$$

From the nature of the transformation function $\omega(\zeta)$, the poles stand at points $(-b_n)$ and hence using the residue of each pole, we have,

$$\phi(\zeta) = \phi_o(\zeta) + \sum_{1}^{4} \frac{a_n}{+b_n} \frac{\bar{\phi}'(b_n)}{\bar{\omega}'(b_n)} \qquad (2.33)$$

$$\psi(\zeta) = \psi_o(\zeta) - \sum_{1}^{4} \frac{a_n}{b_n - \zeta} \frac{\phi'(b_n)}{\bar{\omega}'(b_n)} \qquad (2.34)$$

2.4 DEFORMATION DUE TO HERTZIAN CONTACT LOAD

When a pair of teeth comes in contact and carry load the contact profile suffers from Hertzian deformation at the contact area. The half width of the strip of contact between the meshing teeth is given by Hertz as,

$$b = \left[\left(\frac{4W}{\pi}\right)\left(\frac{\rho_1\rho_2}{\rho_1+\rho_2}\right)\left\{\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right\}\right]^{1/2} \qquad (2.35)$$

where ρ_1, ρ_2 are the radii of curvature of the surfaces in contact and W load in lb.

The resulting deformation of the meshing teeth is given by Weber [5] as

$$z_H = \frac{2\,W\,(1-\nu_1^2)}{\pi \quad E_1}\left[\log\frac{2\,h_1}{b} - \frac{\nu_1}{2\,(1-\nu_1)}\right] + \frac{2\,W\,(1-\nu_2^2)}{E_2}\left[\log\frac{2\,h_2}{b} - \frac{\nu_2}{2(1-\nu_2)}\right] \tag{2.36}$$

The addition of individual deflections of each tooth and the combined contact deformation yields the total deformation from which the total stiffness of the individual pair is determined.

$$z_T = z_G + z_P + z_H \tag{2.37}$$

z_G = Gear tooth deflection

z_P = Pinion tooth deflection

z_H = Hertz deflection

z_T = Total deflection

2.2 CONTACT-RATIO

For a gear-set with contact ratio more than unity, when a pair come in contact there is a certain amount of mis -match because the pair in engagement has suffered a deformation'e'(Fig.2-4). The incoming profile 'z' lags behind by the same amount 'e' and hence the contact starts at a position 's'', earlier than the theoretical position 's'. Similarly the disengagement is post-poned. Effectively the contact ratio is thus increased. This has marked effect on the combined mesh stiffness. The exact

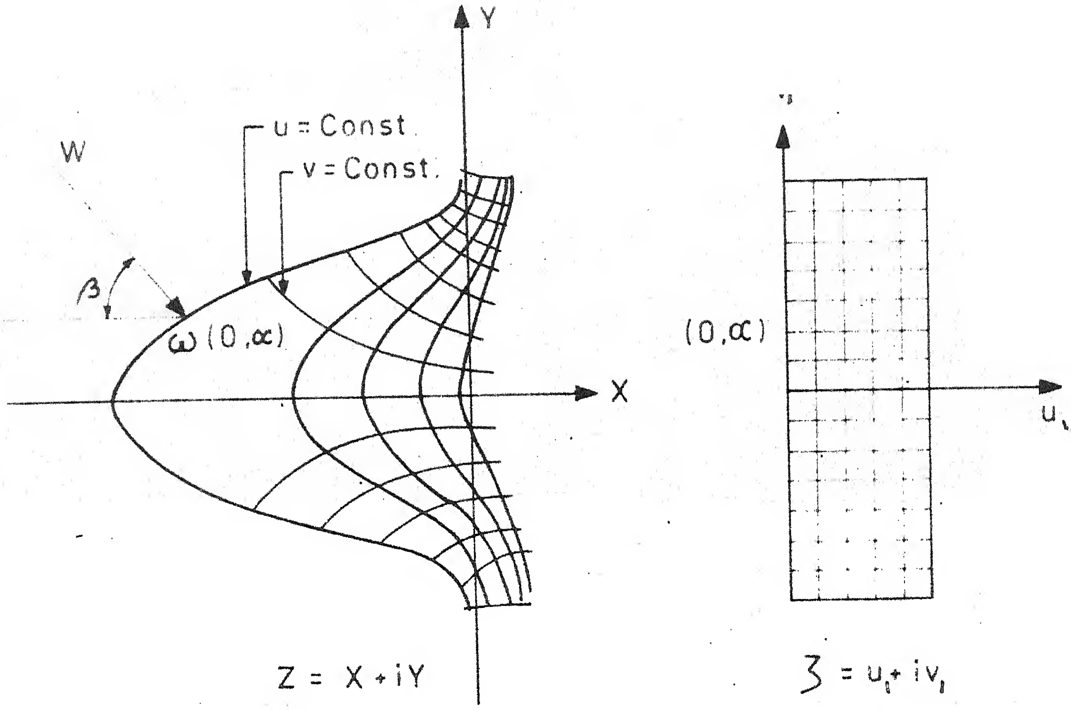


FIG. 2.1. Conformal mapping of semi-infinite region with gear tooth-profile as boundary into semi-infinite region with straight boundary.

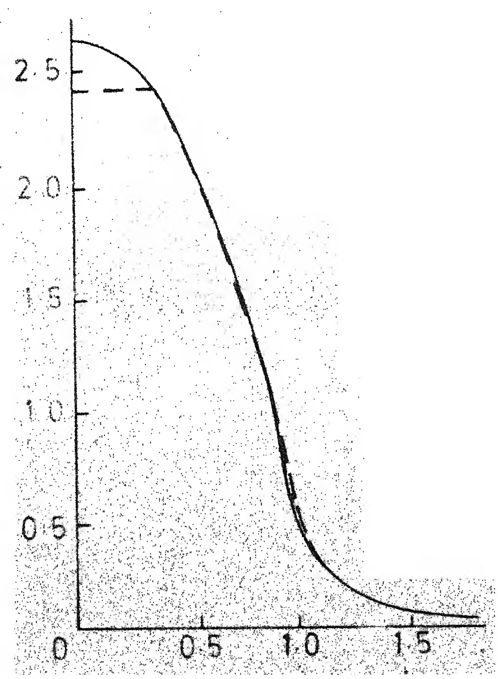


FIG. 2.2. Matching of spur gear profile of 40-teeth full depth, 20° pressure angle.

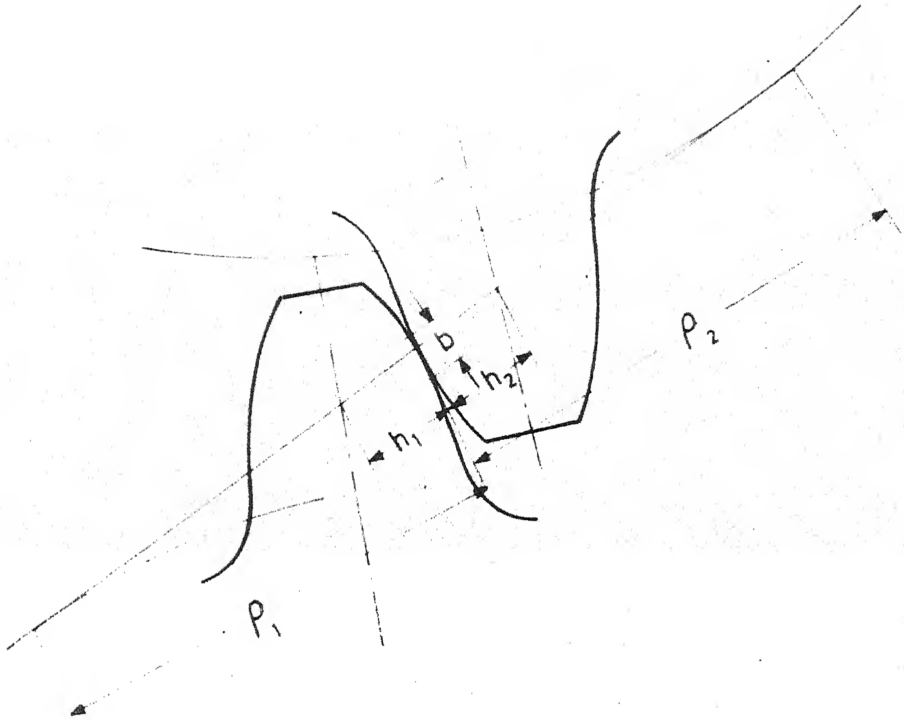


FIG. 2·3 Contact deformation.

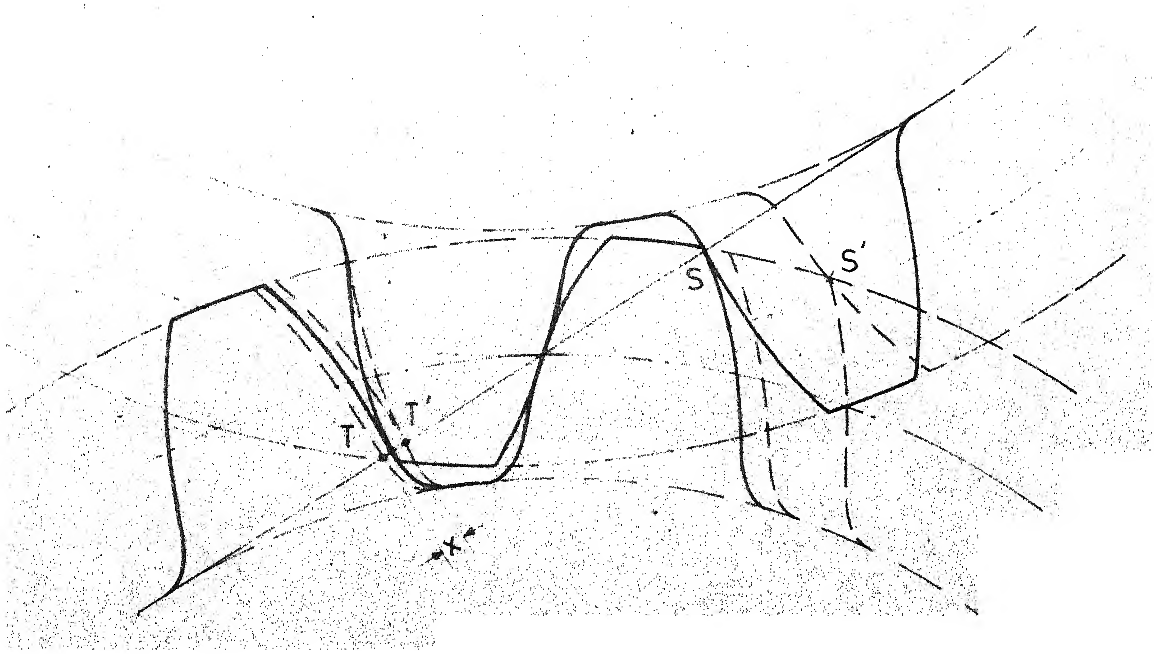


FIG. 2·4. Shift in contact position due to deflection.

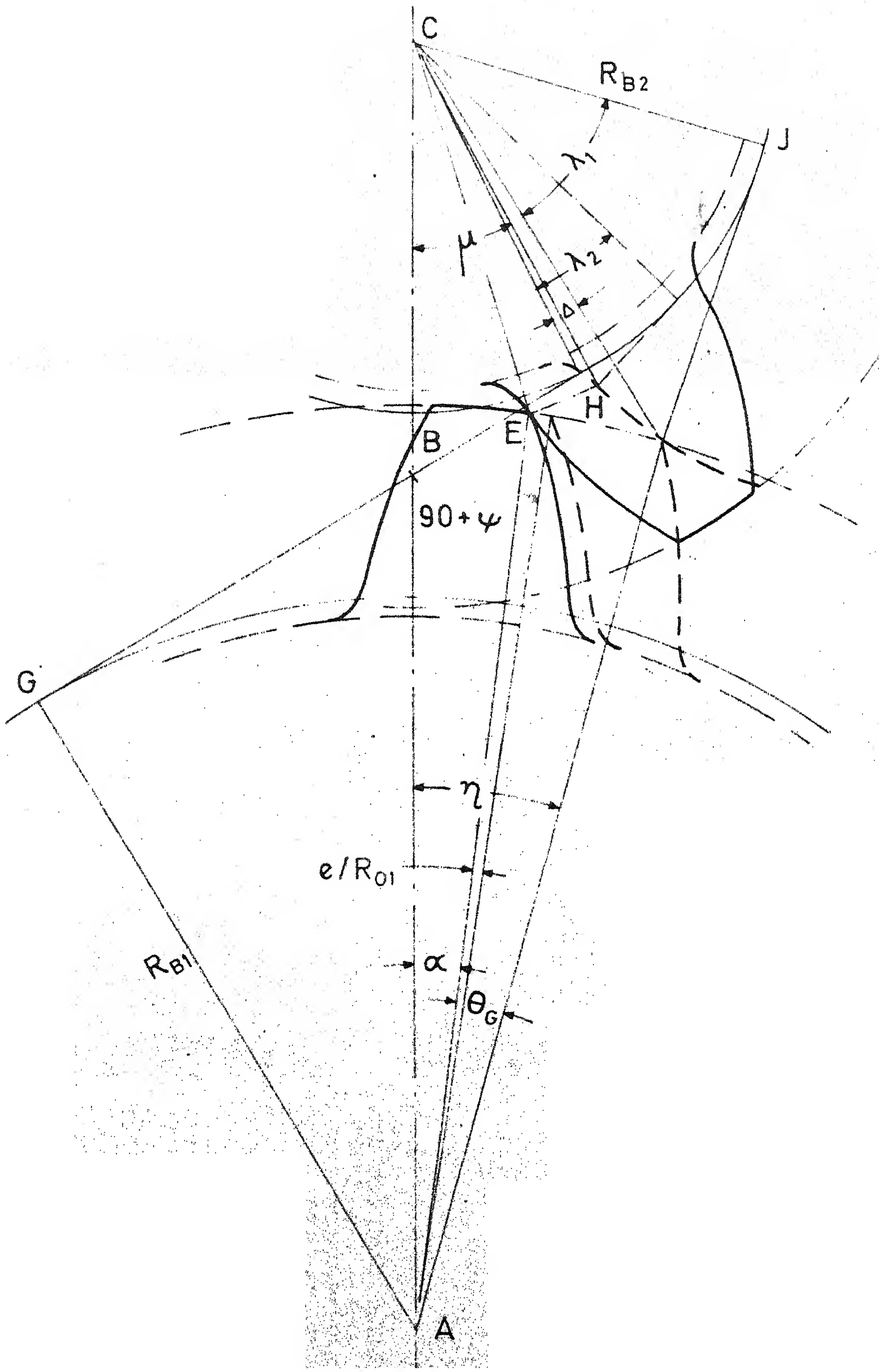


FIG. 2.5 Geometrical conditions for contact with positive error.

positions of engagement and disengagement can be calculated by the geometry as follows 25 :

From triangle BEA in Fig. 2.5

$$\angle BEA = \sin^{-1}\left(\frac{R_1 \sin(90+\alpha)}{R_{10}}\right)$$

$$\alpha = 180 - (90 + \phi) - \angle BEA$$

From triangle ACE,

$$CE = (R_{10}^2 + C^2 - 2R_{10}\, C \cos\alpha)^{1/2}$$

$$\angle BCE = \beta = \sin^{-1}\left(\frac{R_{10} \sin\alpha}{CE}\right)$$

$$\eta = \alpha + \frac{e}{R_{10}} + \theta_G$$

Then

$$DC = (C^2 + R_{10}^2 - 2\, C\, R_{10} \cos\alpha)^{1/2}$$

$$\mu = \sin^{-1}\left(\frac{R_o \sin\alpha}{DC}\right)$$

$$\Delta = \mu - n_G\, \theta_G - \beta \tag{2.38}$$

But by involute geometry

$$\Delta = \operatorname{inv}\lambda_1 - \operatorname{inv}\lambda_2 \tag{2.39}$$

where $\lambda_1 = \cos^{-1}(R_{2b}/DC)$

$\lambda_2 = \cos^{-1}(R_{2b}/CH)$ $(CH = CE)$

and $\operatorname{inv}\lambda = \tan\lambda - \lambda$.

The symbols are to be taken to represent the quantities as shown in Fig. 2.5.

The advance angle θ_G determining the first contact position is obtained by the solution of simultaneous equations (2.38) and (2.39).

4.3 MESH STIFFNESS AND STATIC DISPLACEMENT

Two successive pairs of teeth come in engagement with a phase difference of the base pitch. The displacement characteristics are periodic with a period of the base pitch. The combined mesh stiffness is evaluated by placing the individual stiffness curves at a phase differenceof base pitch. The actual engagement and disengagement point for any tangential load are found out by using the analysis of the section (2.2.) The stiffness of the individual pair in the region beyond the theoretical region is obtained by extrapolation.

From the combined mesh stiffness and the profile error, the static displacement is computed as follows:

$$xs = \frac{W + e_1 k_1 + e_2 k_2}{k}$$

where

W = tangential load

e_1 = combined profile error for the first pair

e_2 = combined profile error for the second pair

k_1 = instantaneous mesh stiffness of the first pair

k_2 = instantaneous mesh stiffness of the second pair

k = instantaneous combined mesh stiffness

Profile error here means the combined error due to geometrical error and total elastic deformation 'e'.

2.4 DYNAMIC RESPONSE

In the dynamic model a pair of gear in engagement is replaced by two masses connected by a spring with time dependent stiffness; the masses being the equivalent masses of the gears placed at the base circle and the stiffness is equal to the combined mesh stiffness. The equation of motion for the system can be formulated as follows :

Symbols :

P_T - total load

P_s - static load

T - shaft torque

T_f - frictional torque

I - mass moment of inertia

M - mass at base radius

K - stiffness

For the dynamic equilibrium of individual gear,

$$I_1 \ddot{\theta}_1 = (P_T - P_s) r_1 + T_{f1}$$

$$\ddot{\theta}_1 = (P_T - P_s) \frac{r_1}{I_1} + \frac{T_{f1}}{I_1} \qquad (2.41)$$

Similarly, $$\ddot{\theta}_2 = (P_T - P_s) \frac{r_2}{I_2} + \frac{T_{f2}}{I_2} \qquad (2.42)$$

θ_1 and θ_2 are having different signs.

But the combined displacement is given by

$$x = -(r_1 \theta_1 + r_2 \theta_2)$$

Therefore

$$\ddot{x} = -(r_1 \ddot{\theta}_1 + r_2 \ddot{\theta}_2) \tag{2.43}$$

Substituting for $\ddot{\theta}_1$ and $\ddot{\theta}_2$, we get

$$\ddot{x} = -(P_T - P_o)\left(\frac{r_1^2}{I_1} + \frac{r_2^2}{I_2}\right) - \left(\frac{T_{f1}}{I_1} + \frac{T_{f2}}{I_2}\right)$$

$$\text{i.e. } \ddot{x} + \left(\frac{T_{f1}}{I_1} + \frac{T_{f2}}{I_2}\right) + P_T\left(\frac{r_1^2}{I_1} + \frac{r_2^2}{I_2}\right) = P_s\left(\frac{r_1^2}{I_1} + \frac{r_2^2}{I_2}\right) \tag{2.44}$$

But $\quad \dfrac{I_1}{r_1^2} = M_1$

$$\frac{I_2}{r_2^2} = M_2$$

so putting $\dfrac{1}{M_e} = \dfrac{1}{M_1} + \dfrac{1}{M_2}$

and since the second term represents the frictional resistance, replacing it by the damping term $f\,\dot{x}$ we get

$$\ddot{x} + f\,\dot{x} + \frac{P_T}{M_e} = \frac{P_s}{M_e} \tag{2.45}$$

f is the damping coefficient.

If e_1 and e_2 are profile errors, then the total force is given by

$$P_T = k_1(x - e_1) + k_2(x - e_2) \tag{2.46}$$

Then equation of motion becomes

$$\ddot{x} + f\,\dot{x} + \frac{k_1}{M_e}(x - e_1) + \frac{k_2(x - e_2)}{M_e} = \frac{P_s}{M_e}$$

Simplifying

$$\ddot{x} + f\,\dot{x} + \frac{k_1 + k_2}{M_e}\,x = \frac{P_s + k_1 e_1 + k_2 e_2}{M_e}$$

$$= \frac{k_1 + k_2}{M_e}\;\frac{P_s + k_1 e_1 + k_2 e_2}{k_1 + k_2} \qquad (2.47)$$

By putting $\omega^2 = \frac{k}{M_e} = \frac{k_1 + k_2}{M_e}$

$$xs = \frac{P_s + k_1 e_1 + k_2 e_2}{k_1 + k_2}$$

xs represents the static displacement.

Then we get

$$\ddot{x} + 2\omega\zeta\,\dot{x} + \omega^2 x = \omega^2 x_s \qquad (2.48)$$

where ζ = ratio of actual damping to initial damping

$= f/2\sqrt{k\,M_e}$

The load carried by each pair is evaluated by utilizing the principle of two springs in parallel due care being given to the errors. The required equations can be shown to be

$$P_1 = k_1 (x - e_1) \qquad (2.49)$$

$$P_2 = k_2 (x - e_2) \qquad (2.50)$$

2.5 COMPUTATION-ALGORITH

Once the analytical formulation to evaluate the static deflection, mesh-stiffness, the error in action, the relative displacement of the pinion and the gear, the dynamic response and dynamic load of the gear-pinion system is complete, these characteristics are computed for different sets of the gear and the pinion. The whole procedure is explained in steps as follows:

Static-deflection-calculation :

Step 1 The mapping coefficients for the pinion and gear are calculated from the values of the coefficients given by Borenet[ ]. Any intermediate value is found by interpolation. Using these coefficients, the transformation function is evaluated and the corresponding coordinates of the profile and central-line of the gear rooth in z-plane and ζ -plane are calculated. The divided difference tables for the respective co-ordinates are evaluated.

Step 2 From the gear geometry the relative positions of the centres of the fixed system and tooth system of co-ordinates are fixed.

Step 3 The engagement range for the set under consideration is evaluated. The path is divided into 100 equal parts and the radii of the curvature of the profiles at contact at these points are calculated.

<u>Step 4</u> At each of the successive points considered above, the co-ordinates of the contact point and the central point, at which the path of contact cuts the central line of the tooth are calculated. The corresponding tooth system coordinates are calculated by coordinate transformation. Then the coordinates of these points in the mapped plane are found by interpolation. With the load acting along the contact line at the contact point the deflection of the central point in the z-plane is calculated by making use of the equation (2.21). The stress functions contain their derivatives at pole points(b_n). These are evaluated by differentiating the stress function and substituting the values of b_n. The resulting four equations along with their conjugates form the eight simultaneous equations, which are solved by using the Gause-Seidel elimination technique. The beam deflections of the wheel-tooth and pinion tooth are calculated separately. Knowing the radii of curvature of the profiles in contact, the Hertzian deformation is evaluated by using the Webber's modified formula (2-36). The sum of all these components gives the total deflection, from which the stiffness of the tooth pair in contact at the position can be calculated.

Mesh-stiffness and Dynamic Response

Step 1 From the deflection data the stiffness characteristics of a pair of teeth, from the beginning of the contact to the end of contact for the gear set chosen are evaluated. The stiffness curves for the consecutive pairs are placed at a phase difference of base pitch and the combined mesh stiffness data are calculated. The increase in contact ratio is taken care of by finding the actual beginning of contact and the end of contact. The individual stiffnesses in this region are calculated by extrapolation. In calculating the actual point of contact, the tooth modifications and manufacturing errors are super-imposed on the error due to the deflection of the pair already in engagement.

Step 2 The error in action is made of two components namely the difference in the deflections of the pairs in contact and manufacturing errors. In the conjugate contact-region, the deflections of the two pairs are equal and hence the errors are made of only the manufacturing errors. In the non-conjugate action-region, the error due to the differential displacements are evaluated by the geometry for tip contact.

Step 3 The deflection-curves are evaluated for the static loading conditions using the equation (2.40).

<u>Step 4</u> The equation of motion (2.49) is solved by the classical Runge-Kutta Method by using the stiffness and static deflection data at regular intervals. The total number of intervals taken are hundred. The corresponding time interval is calculated by finding the time required for covering one pitch, depending upon the speed.

<u>Step 5</u> The dynamic loads carried by the pairs are calculated by using the equations (2.50). The maximum values for each pair and the corresponding position are found. The greater of the two maximum values is the greatest dynamic load and the dynamic factor is calculated.

The dynamic factors for different combinations of gears and speeds are evaluated.

COMPUTATIONAL ALGORITHM

DEFLECTION CALCULATION

READ DATA
Profile mapping coeff.
No. of teeth in the gear set

Mapping coefficients for the gear set under consideration. Evaluate the tooth profile cordinate and the central line-data in z-plane and ζ-plane.

Form and store the divided difference tables for the corresponding co-ordinate for the profiles and central lines

Calculate the tooth dimensions. Locate the origin of the tooth-system with respect to the fixed co-ordinate system of the respective gear

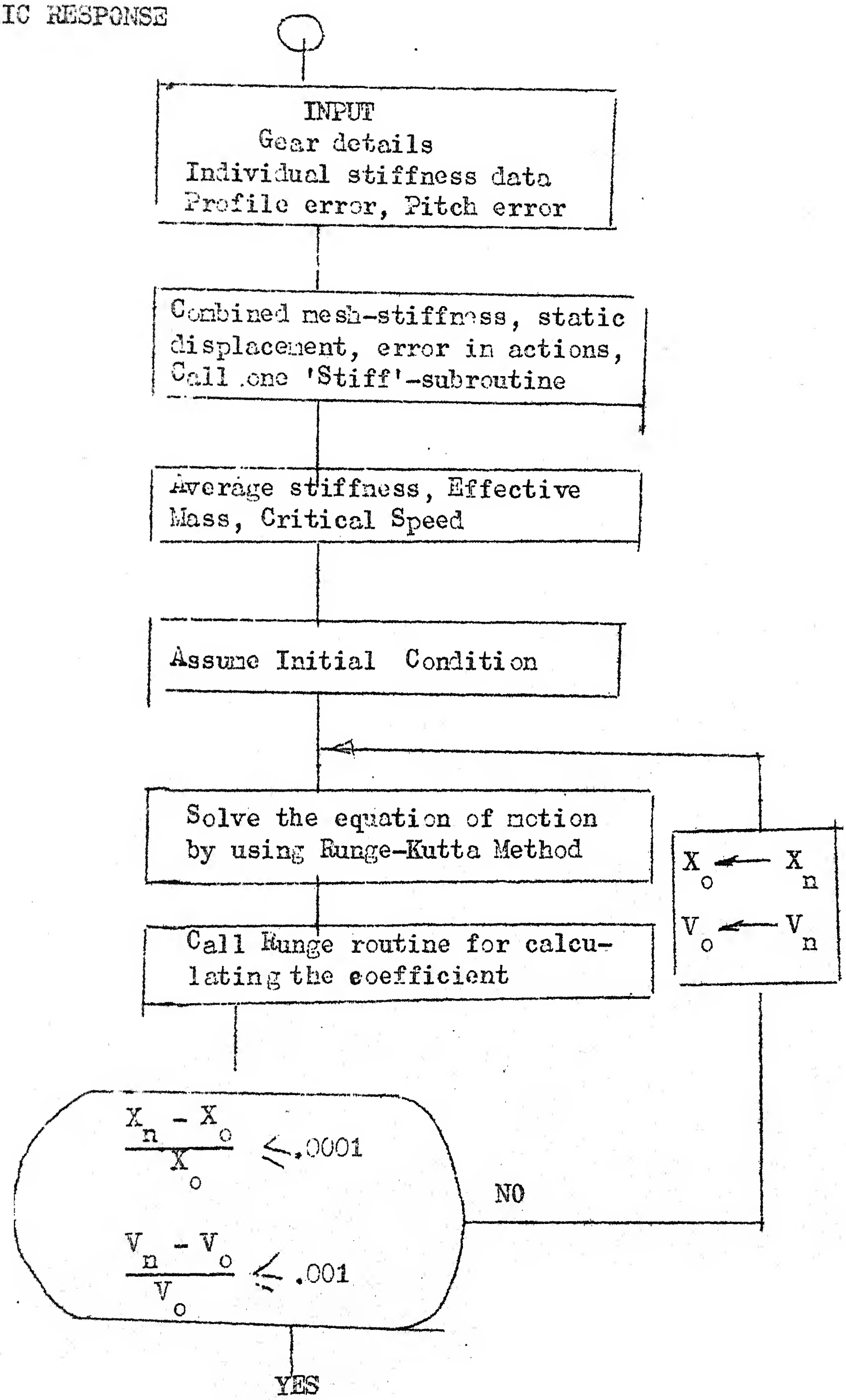
Calculte the range of engagement and fix the intervals for the deflection calculations

For each position of contact, find the co-ordinate in tooth system by co-ordinate transformation and interpolate the value to get the corresponding co-ordinates in ζ-plane.

CALL THE deflection subroutine to calculate the deflection at the point, for each gear-calculate the Hertezian contact deformation, and the total deflection.

(B)

DYNAMIC RESPONSE



1

2

Call Dynamic Load at each position. Find individual maximum and over maximum Dynamic Factor

5

ADDITIONAL ROUTINE FOR PITCH ERROR

ANI

4

$X_o \leftarrow X_n$

$V_o \leftarrow V_n$

Error in action, Mesh-stiffness, Displacement for Two Mesh periods

Solve the Dynamic equation for Two Mesh-periods

2

TABLE 1

MAPPING COEFFICIENTS

Pressure Angle 20°

No. of Teeth / Coeff.	20	30	40	60	80	150
b_1	.005	.005	.005	.005	.005	.005
b_2	.025	.025	.025	.023	.023	.022
b_3	.127	.127	.127	.125	.124	.124
b_4	.490	.487	.483	.470	.455	.450
a_1	.0024	.0024	.0024	.0024	.0024	.0024
a_2	.017	.016	.015	.013	.012	.012
a_3	.130	.127	.124	.115	.112	.110
a_4	.220	.250	.275	.325	.335	.340

3. RESULTS AND DISCUSSIONS

The results of computation and the discussion of the results are given below. Some typical gear tooth combinations are taken. The diametral pitch of the teeth is 4/inch in all the cases. The material of both the gears is steel having E to be equal to 30.10^{6} lb/in^{2} and to be 0.3. The teeth have standard involute form and the pressure angle is taken to be 20° degrees.

3.1 COMBINED STATIC DEFLECTIONS OF THE CONTACTING PAIR OF TEETH :

The static deflections of a pair of teeth in contact are found by using the complex variable method equations 2.21-2.34 . The following gear tooth combinations are taken : 20P-30G, 20P-40G, 20P-50G, 30P-30G, 40P-40G, and 50P-50G, 60P-60G and 70P-70G. XP-YG means that the pinion has X teeth and thegear y teeth. Some of the results are shown in Figs. 3.1, 3.2, 3.3. There is a gradual reduction in combined deflection as the contact point approaches the pitch point and it increases as the point moves away. The deflection characteristic is similar to those we got by energy method . The exact form of the combined deflection changes with number of teeth (reduces with increased number) and normal load on gear tooth.

3.2 MESH-STIFFNESS AND RELATIVE STATIC DISPLACEMENT OF THE TEETH :

For a particular pair of teeth (one of the pinion and the other of the gear) in contact, it is obvious that the stiffness characteristics will be droping down from approximately the pitch

point. As the contact ratio of the spur gears, in general, is quite appreciable (about 1.7), for most of the contact path two pairs will be in contact. The resultant combined mesh-stiffness of the gear system for different combinations of teeth is shown in Fig. 3.5. It is seen that as in the neighbourhood of the pitch point only one pair of teeth is transmitting power and because of the combined effect, the stiffness is least in this zone. At the transition period when the contact is spreading from two pairs of teeth to one pair or when the contact is shifting from one pair to two pairs, there should be a sudden change in the combined mesh-stiffness. But this effect at the transition stages somewhat smoothens out because of the change of the contact ratio due to deflections of the teeth. In this dissertation the effect of impact has not been considered.

As the variation of the tooth deflection for different angles of rotation of the pinion is not constant, the transmission error also varies with the rotation of the pinion. Transmission error can be experimentally measured by marking the gear and pinion under no-load position at a coincidental point and observing the relative displacements of these marks when the gears are in motion under load . The transmission error curve for 20P-30G is given in Figure 11.

DYNAMIC RESPONSE

The boundary value problem (equations 2.49 and 2.50) with damping factor of .1 of critical, is solved by assuming the initial values of the displacement and velocity as

$$X_1 = Xs_1$$

$$V_1 = (Xs_2 - Xs_1)/\Delta T$$

ΔT the interval size. The time T of one cycle, i.e. the time needed for the contact point from the pitch point of one tooth to that of the next tooth is divided into hundred equal divisions. Each of these divisions is equal to Δ T.

The final values are compared with initial values and the following convergence criteria are applied.

$$\frac{X_{100} - X_1}{X_1} \leqq .001$$

$$\left(\frac{V_{101} - V_1}{V_1}\right) \leqq .01$$

The convergence to the desired solution has occured in about 25 iterations. The dynamic response at 500 RPM for 20P-30G is shown in Fig. 3.6. It shows peak values whenever a pair enters or leaves the contact zone. In addition to these high peaks, smaller peaks also occur in between because of the variation of individual tooth stiffness.

The dynamic load curve for 20P-30G combination is shown in Fig. 3.7. It shows peak values at the transition stages. The load on a pair has a maximum value near the pitch point. Many

investigators after conducting experiments observed that near the pitch point the pitting phenomenon was most prominent and it is accepted that when the contact is near the pitch point, the gears experience the worst loading condition. This has been substantiated by the present investigation also. The worst dynamic loads occur when the contact is near the pitch point. Location of the exact spot where worst loading occurs is not feasible either by experimental or theoretical means. The author feels that as the kinematic, geometric and load characteristics do not change appreciably in the neighbourhood of the pitch point, the pitch point itself can approximately be taken as the point representing the worst.

In Fig. 3.7 dynamic load of an individual pair of teeth in contact is shown against the location of the contact point from one pitch point to the next one. The same result is given in Fig. 3.8 to show how the dynamic load varies on this pair of tooth as it comes in contact and leaves contact. The dynamic load factor considering the maximum peak load has been computed and is shown in Fig. 3.9. As expected the factor varies with speed. It increases upto the critical speed of the set. Beyond this it comes down and tends to be constant as the speed increases. The dynamic load factor for speeds well above the critical speed does not vary much for different gear sets. The maximum dynamics-load factor varies for different sets. The highest value is about 2.86. The value is quite comparable to those found experimentally and predicted by some investigators.

Kohler [15] has concluded from his test results that irrespective of error, the dynamic load never exceeded three times the design load. The values calculated by Buckinghams equation [11] are shown in Table 12. They are also below the critical value of 3. Harris states that for modified tooth-gears running at the design load, the factor will have a maximum value of two. There has not been universally accepted rating formula for the dynamic factor.

TABLE 2.

DYNAMIC LOAD FACTORS

Tangential Load 1000 lb.

S.No.	Gear Set	Speed RPM	Dynamic Load (lb)	Dynamic Factor P_T/P_s
1	30P- 30G	500	2859	2.86
2		1000	1200	1.2
3		1500	1559	1.559
4		2000	1666.0	1.66
1	40P- 40G	500	1665	1.6
2		1000	1229	1.229
3		1500	1350	1.358
4		2000	1650	1.650
1	50P- 50G	500	1268	1.268
2		1000	14060	1.4
3		1500	1634.6	1.63
4		2000	1639	1.64

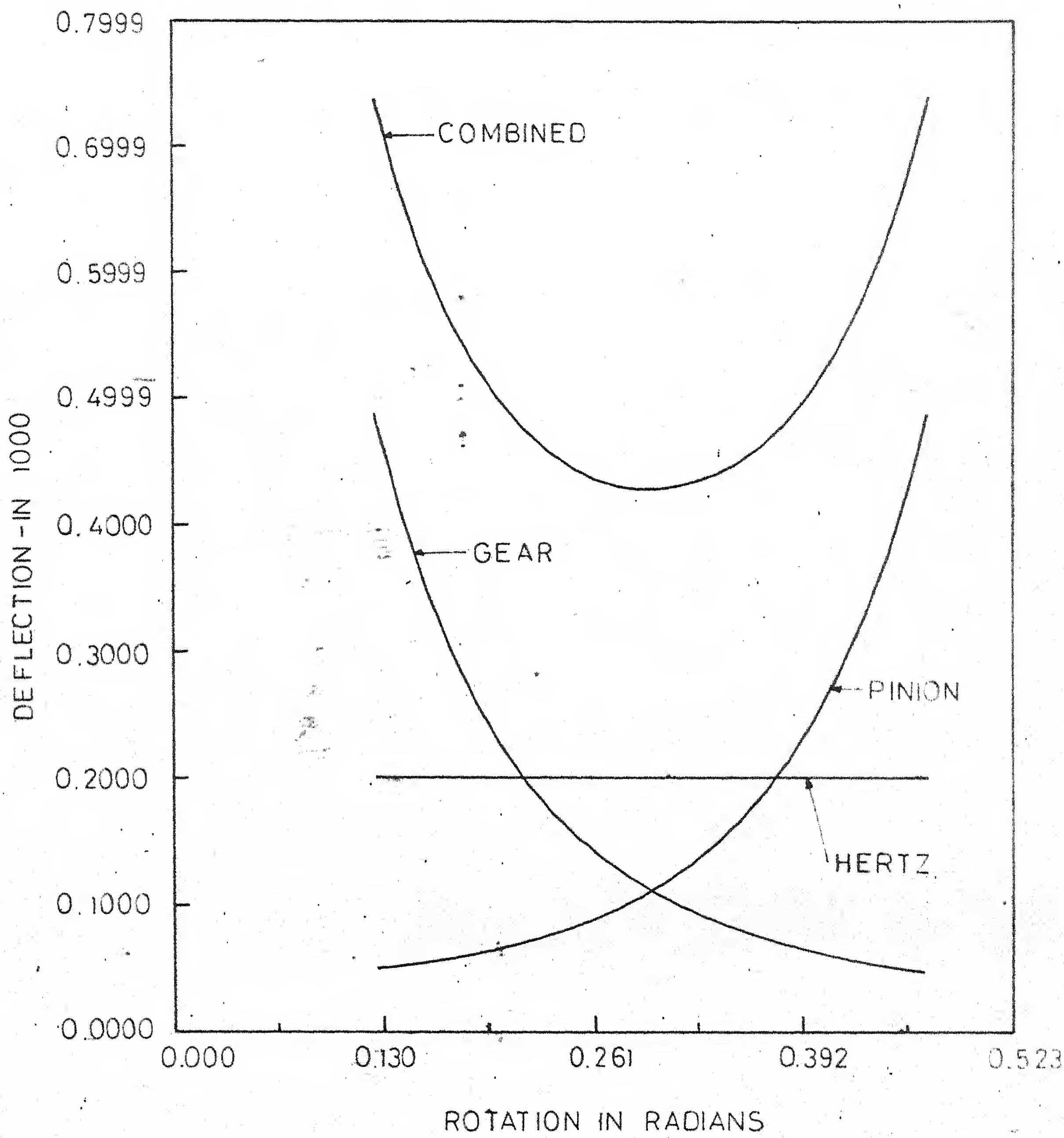


FIG.3.1. STATIC DEFLECTION CURVE.

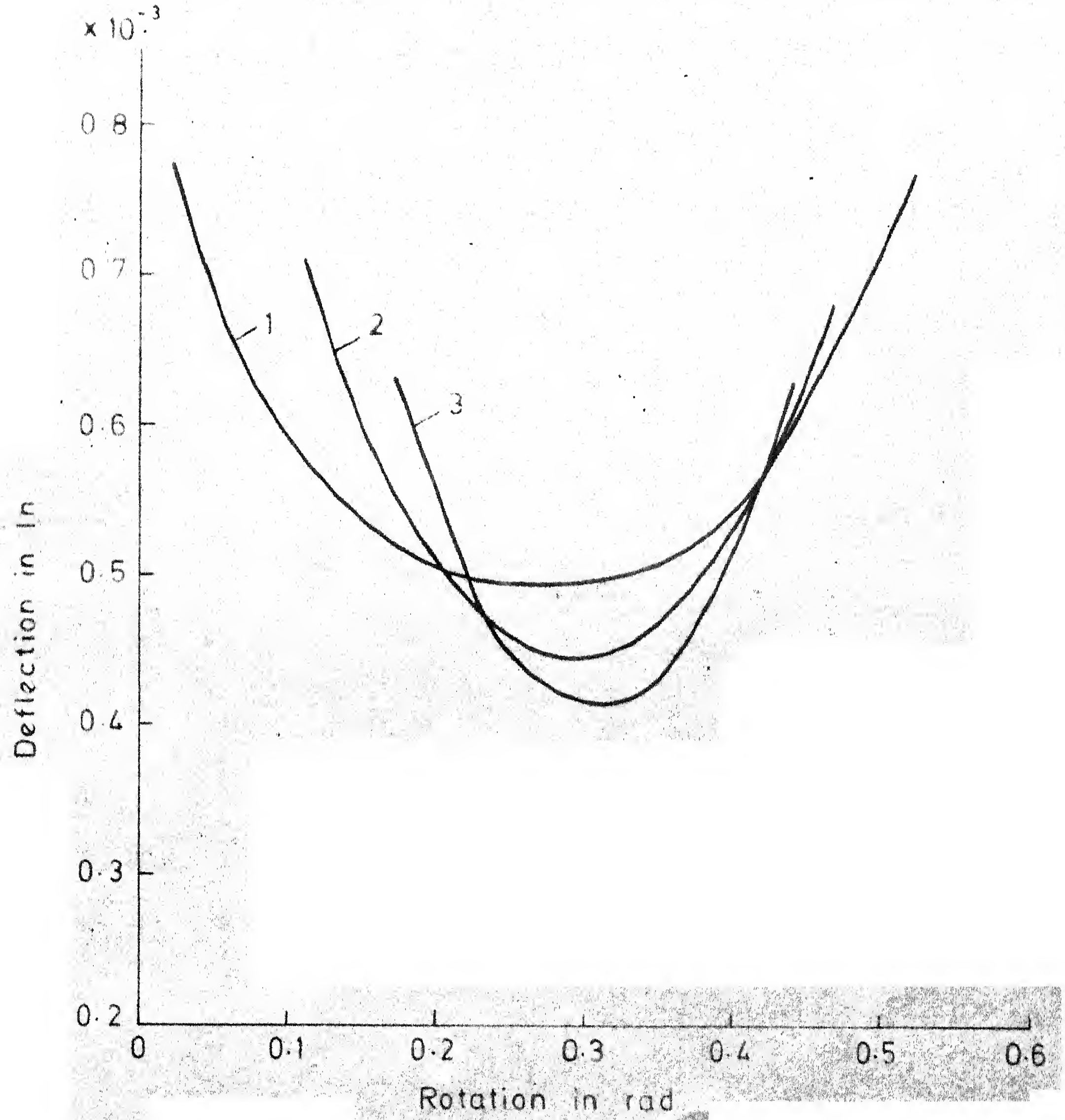


FIG. 3.2a. Deflection for gear ratio 1:1

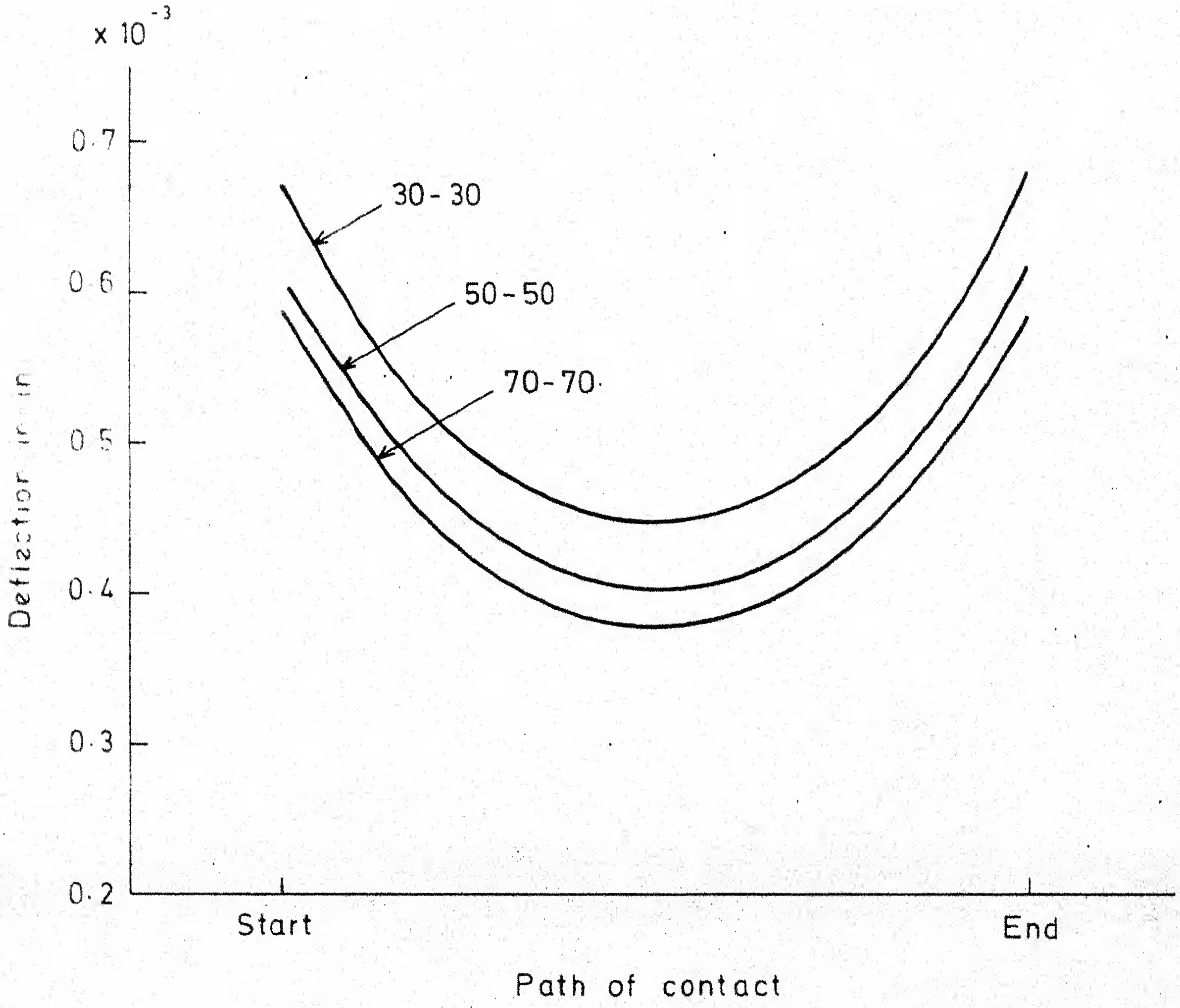


FIG. 3.2b. Deflection for gear ratio 1:1

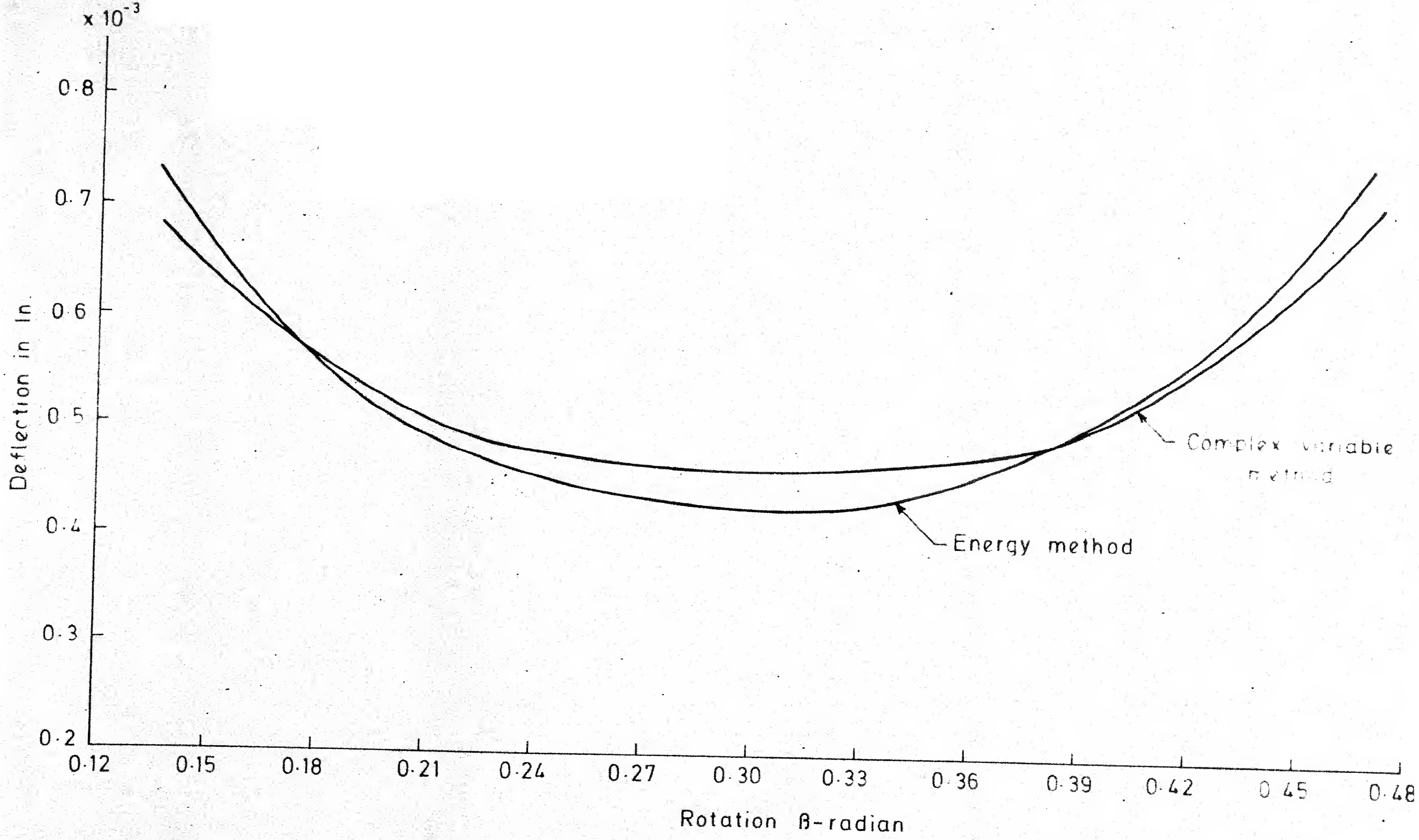


FIG. 3·3. Comparison of energy and complex variable methods.

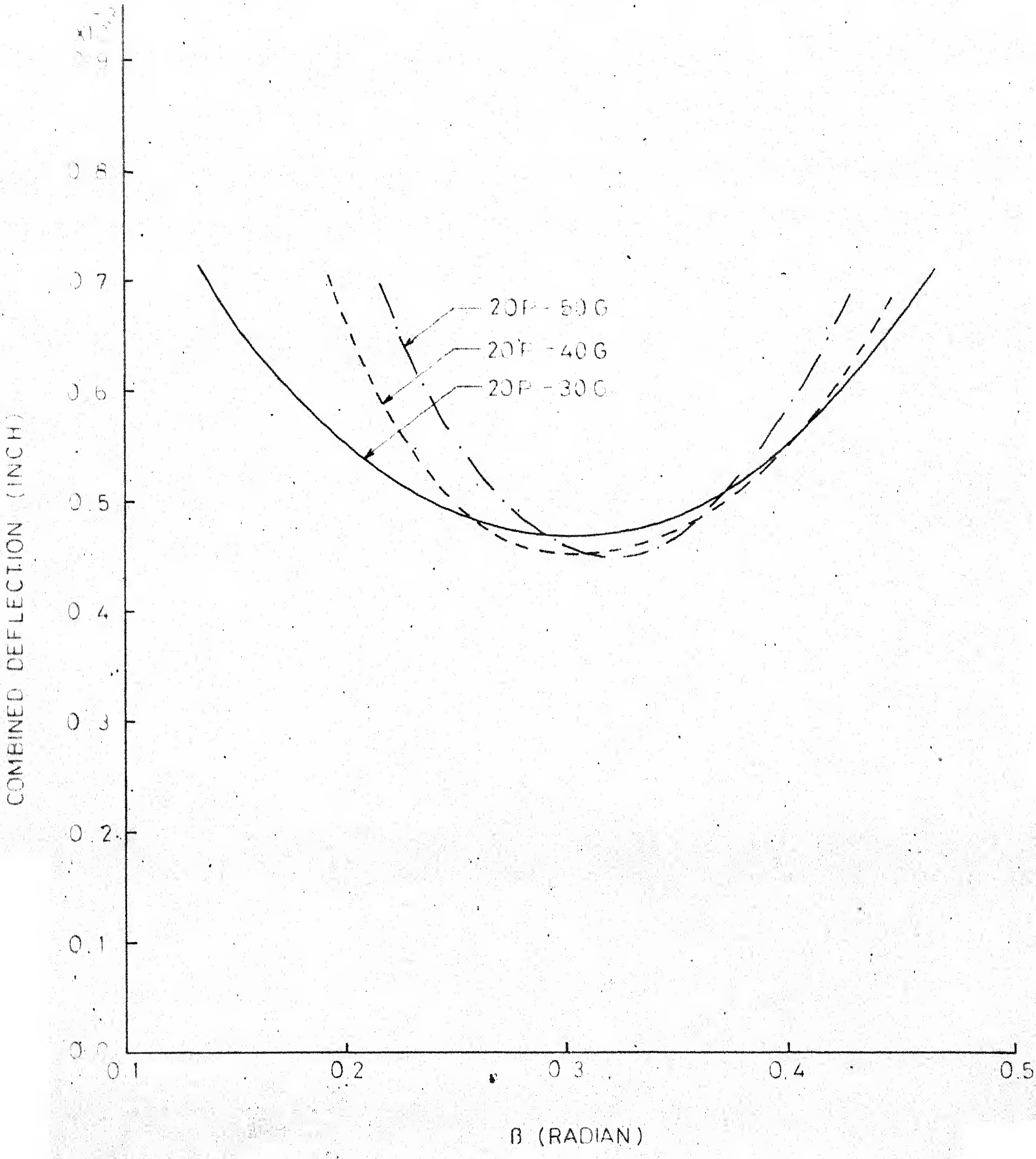


FIG. 3.4. COMBINED DEFLECTION DIFF. GEAR RATIOS.

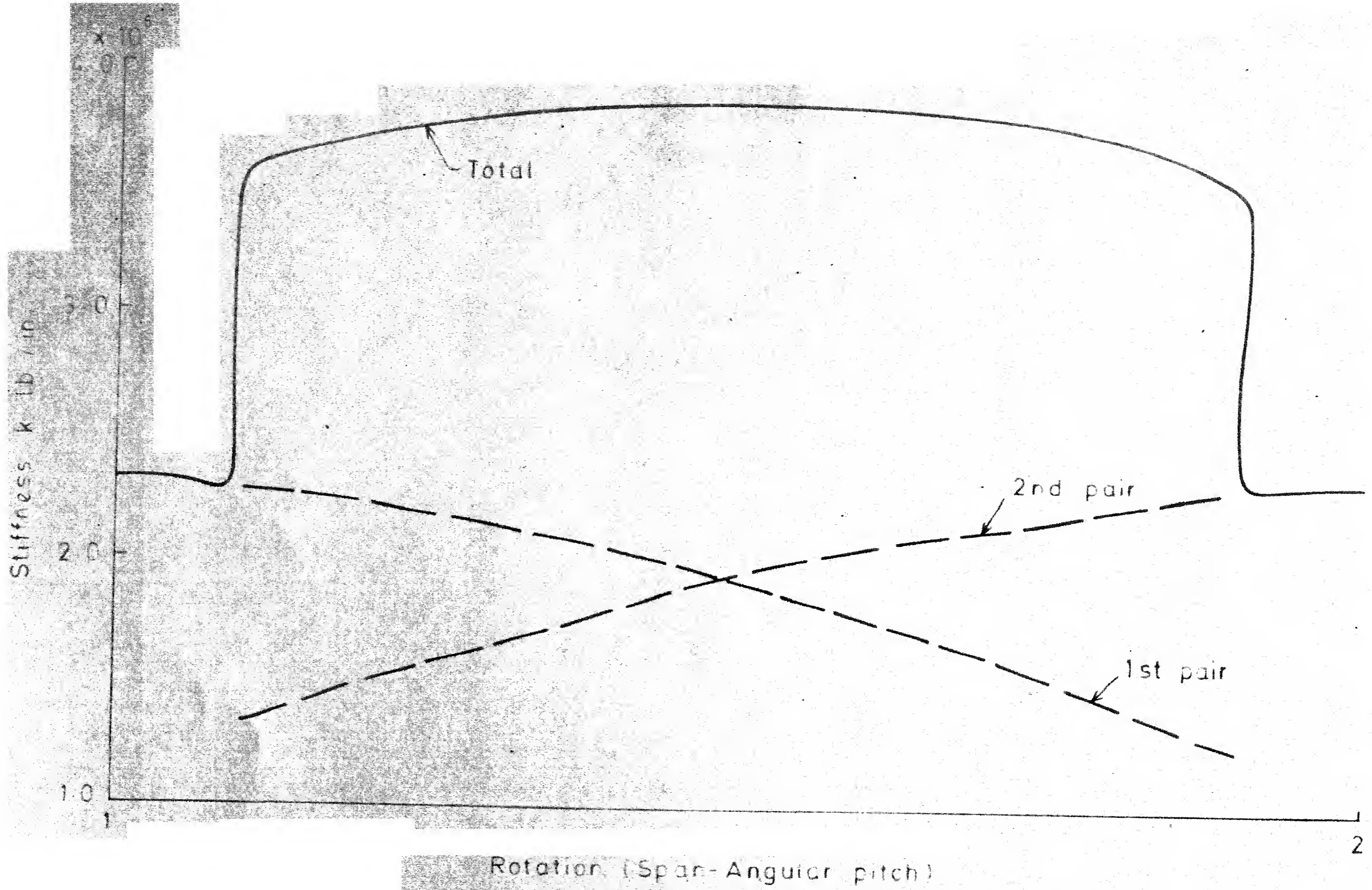


FIG 3.5. Combined mesh-stiffness

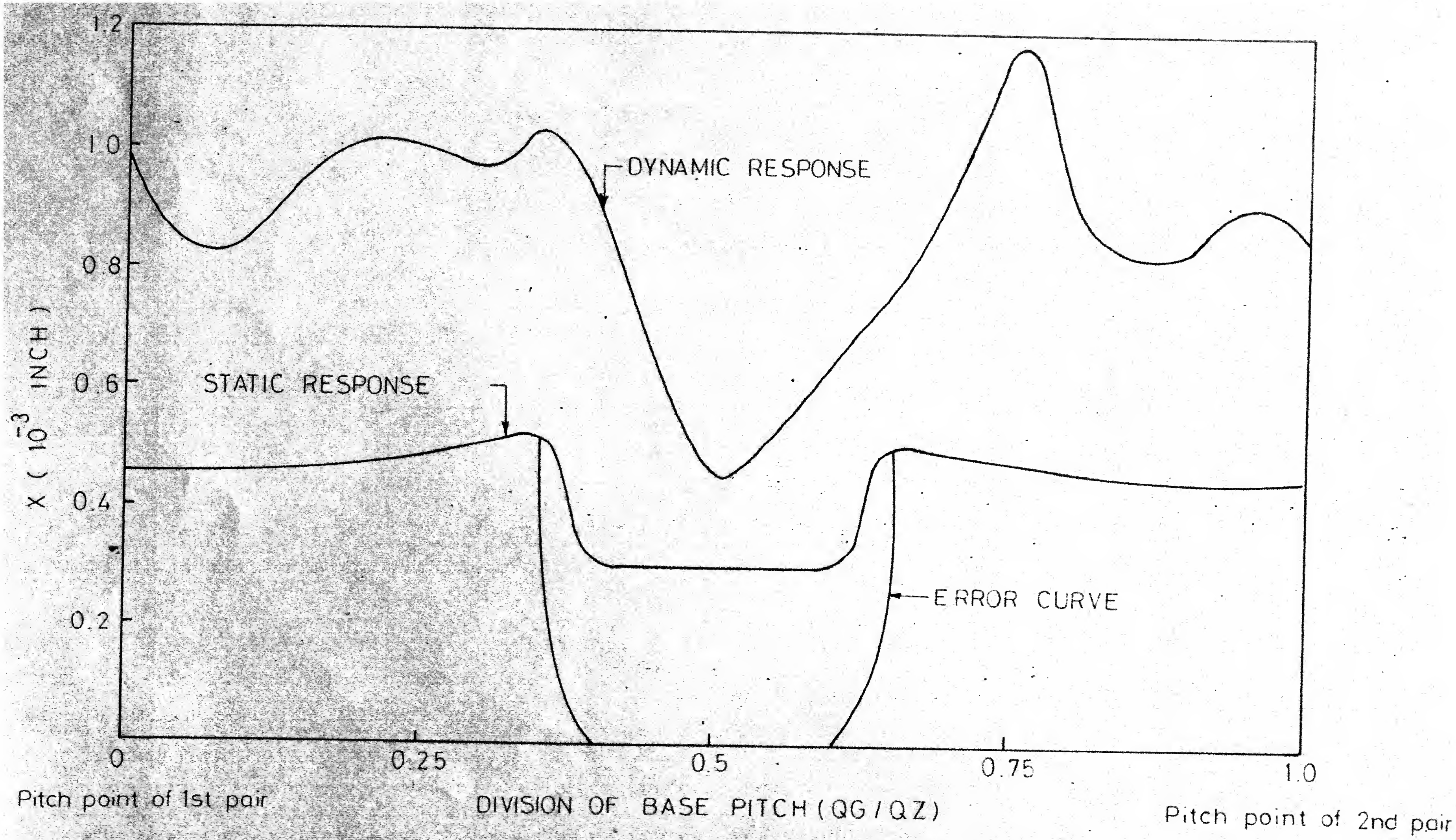


FIG.3.6 STATIC AND DYNAMIC RESPONSE (20P-30G set 500 rpm)

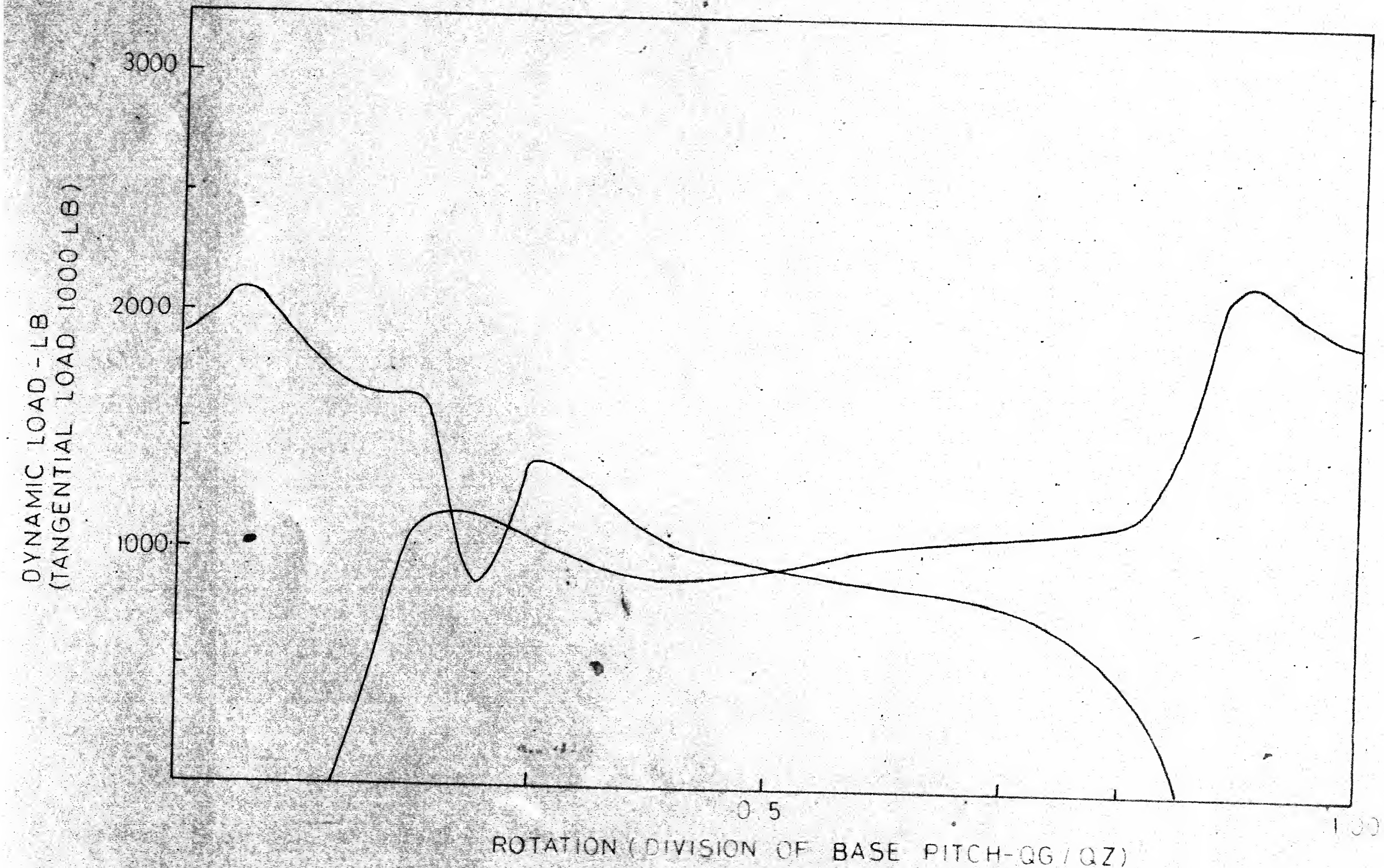


FIG 3.7 INDIVIDUAL DYNAMIC LOAD VARIATION
(20P-30G Set at 500 rpm)

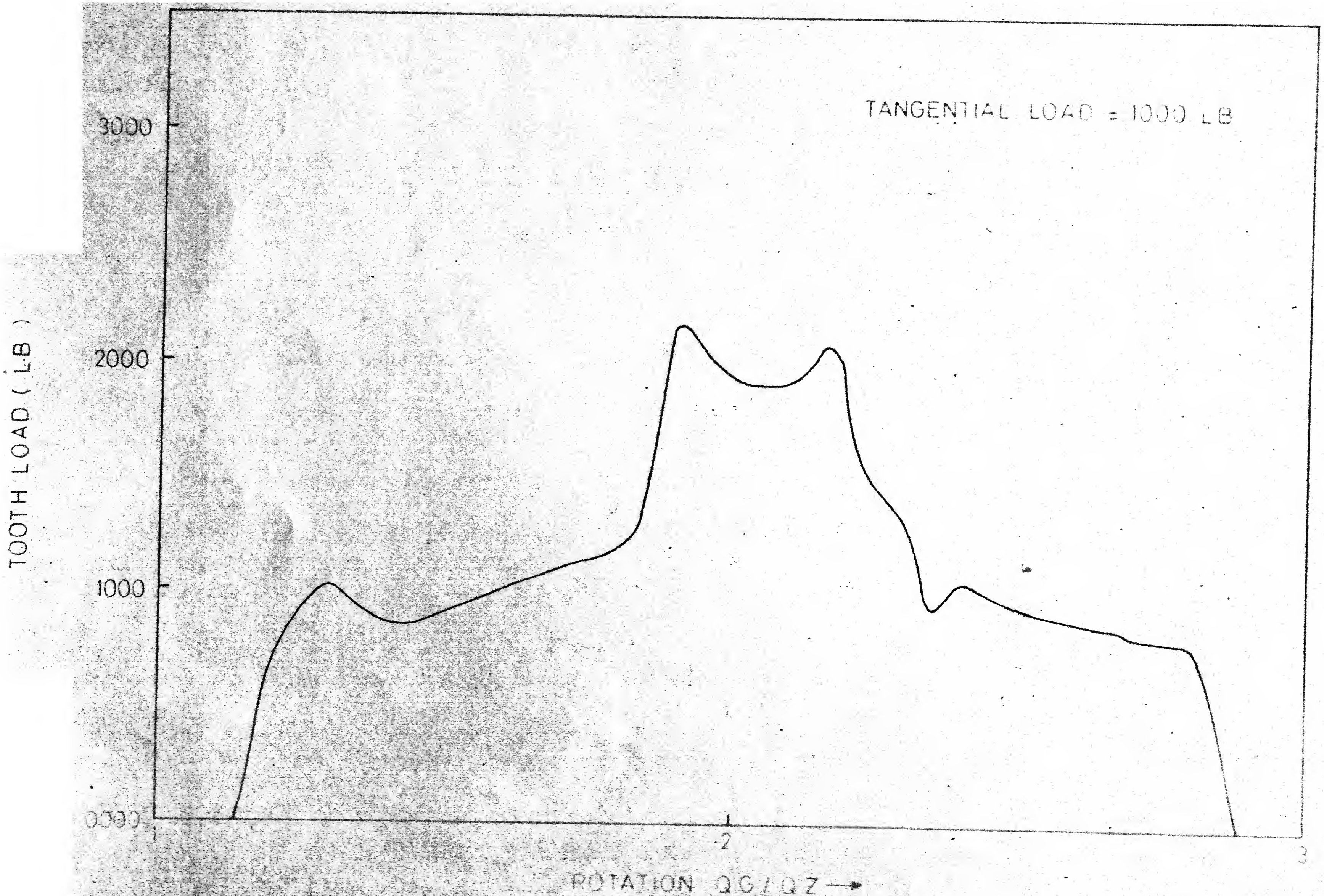


FIG 3 8 DYNAMIC LOAD FOR A TOOTH (20P-30G at 500 rpm)

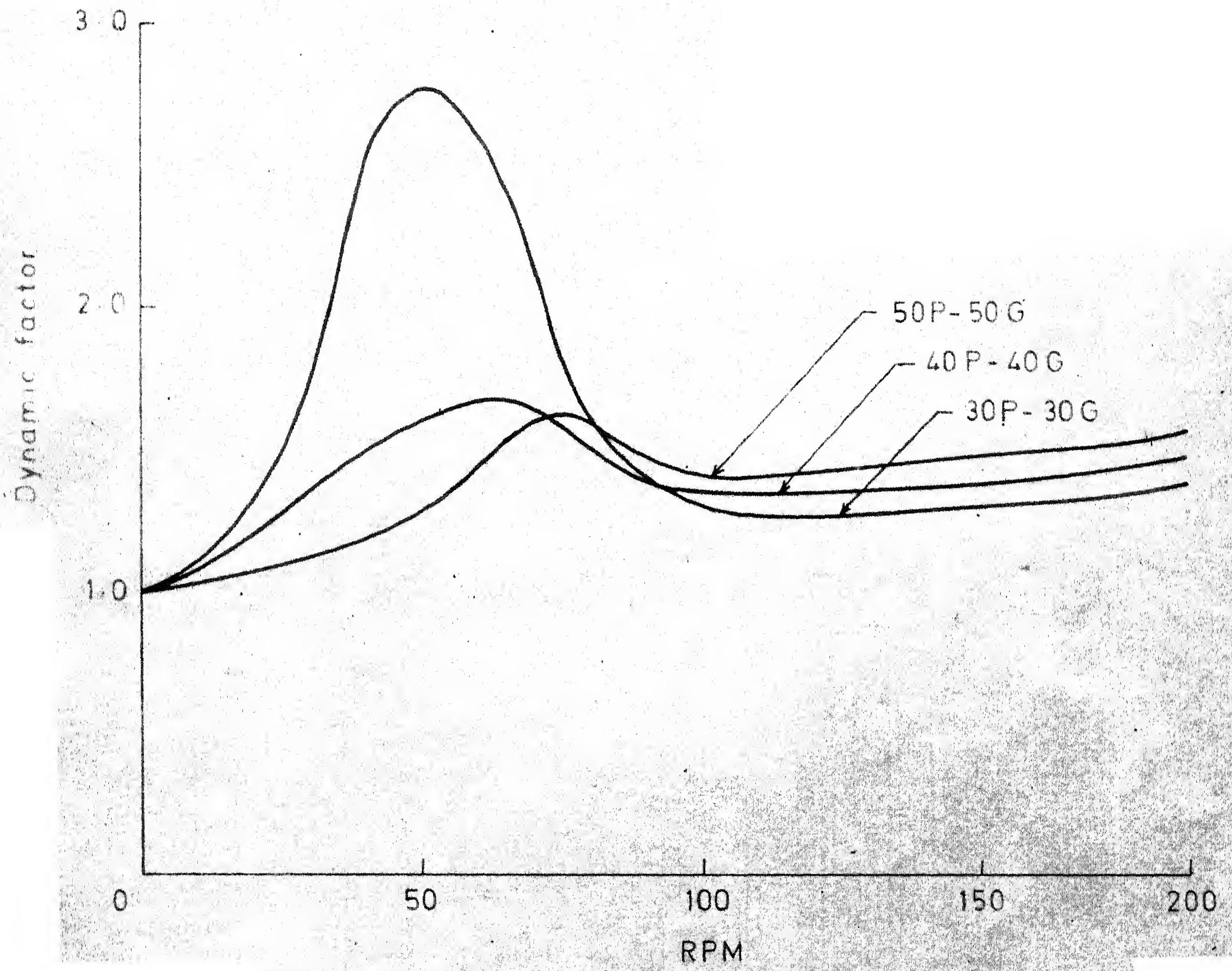


FIG. 3.9. Static and dynamic response.

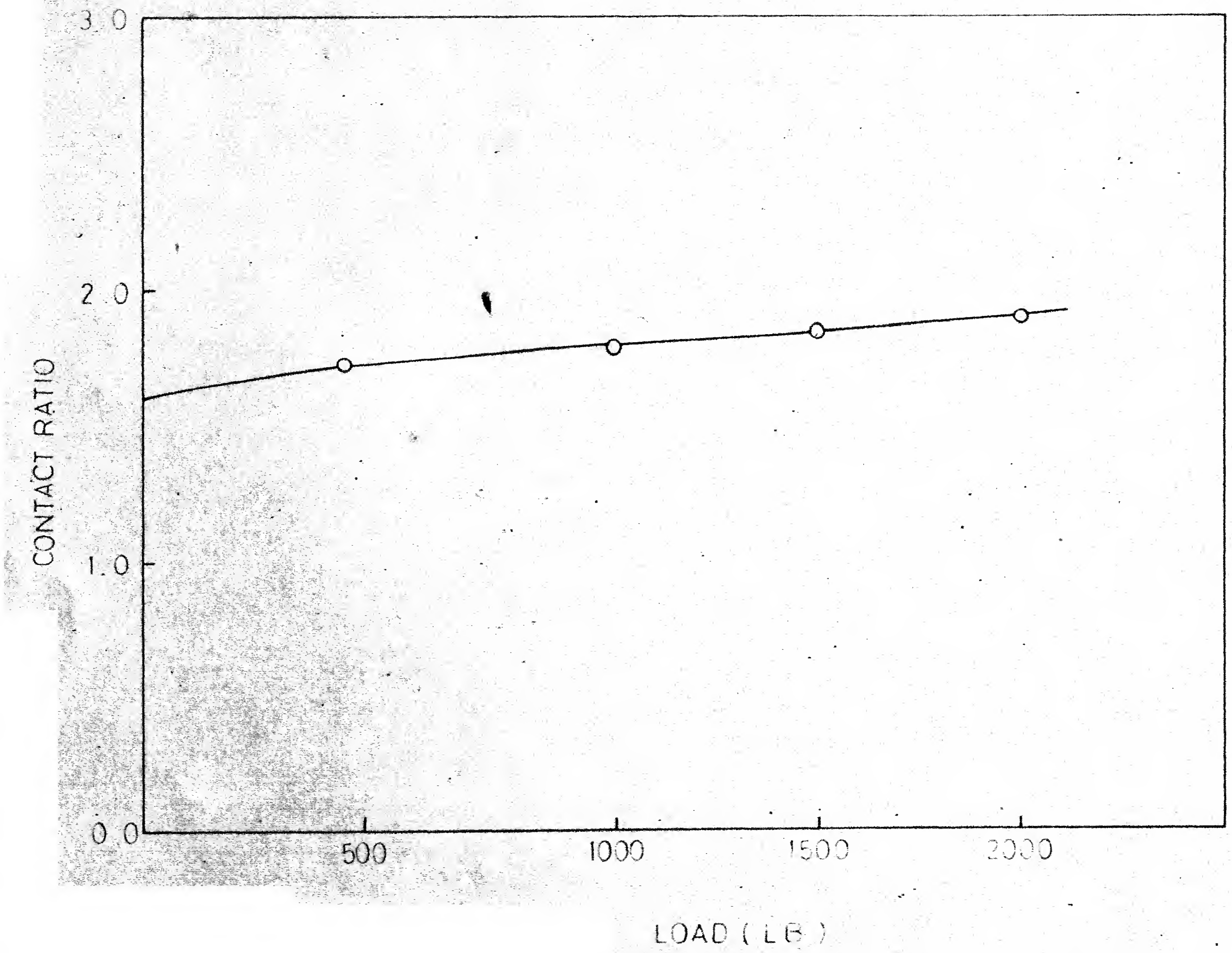


FIG.3.10 INCREASE IN CONTACT RATIO WITH LOAD FOR 20P-30G SET

TABLE - 4

[illegible] FOR Z4= [illegible]0.00000Z2 [illegible]0.00000

BT	ZG	ZP	ZH	ZD
0.1236[illegible]E 00	[illegible].46293128E-03	0.50217498E-04	0.16983784E-03	0.68298661E-03
0.1304775E 00	0.43672905E-03	0.51091466E-04	0.17048959E-03	0.65851004E-03
0.1374[illegible]E 00	[illegible].41[illegible]275E-03	0.53657789E-04	0.17106771E-03	0.63877824E-03
0.1443299[illegible]E 00	[illegible].39554455E-03	0.55520803E-04	0.17158287E-03	0.62264822E-03
0.1512581[illegible]E 00	0.37604740E-03	0.57501262E-04	0.17204368E-03	0.60559233E-03
0.1581[illegible]E 00	0.35851422E-03	0.59588442E-04	0.17245715E-03	0.59055980E-03
0.1651[illegible]863E 00	[illegible].34174743E-03	0.61791845E-04	0.17282903E-03	0.57636831E-03
0.172[illegible]3485E 00	[illegible].32566357E-03	0.64135678E-04	0.17316411E-03	0.56096336E-03
0.17896107E 00	0.30812126E-03	0.66592824E-04	0.17346637E-03	0.54818045E-03
0.1858872[illegible]E 00	0.29814345E-03	0.69200151E-04	0.17373917E-03	0.53608274E-03
0.19281352E 00	0.27952511E-03	0.71958785E-04	0.17398534E-03	0.52546924E-03
0.19973974E 00	0.26645001E-03	0.74859373E-04	0.17420730E-03	0.51551668E-03
0.20666596E 00	0.25355302E-03	0.77948933E-04	0.17440710E-03	0.50590904E-03
0.21359219E 00	0.24099231E-03	0.81173042E-04	0.17458650E-03	0.49675185E-03
0.22051840E 00	0.22959313E-03	0.84610707E-04	0.17474702E-03	0.48895086E-03
0.22744462E 00	0.21903359E-03	0.88208827E-04	0.17488993E-03	0.48213234E-03
0.23437084E 00	0.20855021E-03	0.92027765E-04	0.17501634E-03	0.47559441E-03
0.24129707E 00	0.19861280E-03	0.96036013E-04	0.17512720E-03	0.46977602E-03
0.24822329E 00	0.18946941E-03	0.10027800E-03	0.17522331E-03	0.46497072E-03
0.25514951E 00	0.18082400E-03	0.10472892E-03	0.17530534E-03	0.46085826E-03
0.26207573E 00	0.172555[illegible]E-03	0.10943557E-03	0.17537385E-03	0.45716458E-03
0.26900195E 00	0.164524[illegible]E-03	0.11437186E-03	0.17542931E-03	0.45432554E-03
0.27592818E 00	0.157[illegible]E-03	0.11958992E-03	0.17547207E-03	0.45211640E-03
0.28285440E 00	0.1[illegible]4906E-03	0.12505309E-03	0.17550242E-03	0.45050457E-03
0.28978062E 00	0.14[illegible]E-03	0.13084431E-03	0.17552056E-03	0.44964573E-03
0.29670684E 00	[illegible]	0.13687952E-03	0.17552659E-03	0.44925974E-03
0.30363306E 00	[illegible]E-03	0.14330712E-03	0.17552056E-03	0.44965299E-03
0.31055928E 00	0.12[illegible]E-03	0.14998222E-03	0.17550242E-03	0.45051831E-03
0.31748550E 00	0.11[illegible]7480[illegible]E-03	0.15709441E-03	0.17547207E-03	0.45214128E-03
0.32441173E 00	[illegible]E-03	0.16456171E-03	0.17542931E-03	0.45434744E-03
0.33133795E 00	[illegible].10942264E-03	0.17238253E-03	0.17537385E-03	0.45717901E-03
0.33826417E 00	0.10471796E-03	0.18082101E-03	0.17530553E-03	0.46084430E-03
0.34519039E 00	0.10026771E-03	0.18950163E-03	0.17522330E-03	0.46499265E-03
0.35211661E 00	0.96[illegible]E-04	0.19868879E-03	0.17512720E-03	0.46984448E-03
0.35904283E 00	0.92[illegible]E-04	0.20863890E-03	0.17501633E-03	0.47567841E-03
0.36596905E 00	0.8[illegible]E-04	0.21905669E-03	0.17488992E-03	0.48215295E-03
0.37289528E 00	0.[illegible]E-04	0.22970644E-03	0.17474701E-03	0.48906321E-03
0.37982149E 00	0.[illegible]E-04	0.24115554E-03	0.17458649E-03	0.49691422E-03
0.38674771E 00	0.77[illegible]E-04	0.25[illegible]E-03	0.17440709E-03	0.50591312E-03
0.39367393E 00	0.7[illegible]E-04	0.26669841E-03	0.17420729E-03	0.51576135E-03
0.40060015E 00	[illegible]E-04	0.27[illegible]E-03	0.17398533E-03	0.52562400E-03
0.40752638E 00	[illegible].69[illegible]E-04	[illegible].29[illegible]E-03	[illegible].17373915E-03	0.53647076E-03
[illegible].41445258E 00	[illegible]E-04	[illegible].30[illegible]E-03	[illegible].17346686E-03	0.54828700E-03
0.42137880E 00	[illegible]E-04	0.32[illegible]E-03	0.17316410E-03	0.56158020E-03
0.428[illegible]	[illegible]	[illegible].34[illegible]1391E-03	[illegible].17282902E-03	[illegible].57623335E-03
[illegible].43[illegible]	[illegible]	[illegible].35957375E-03	[illegible].17245713E-03	[illegible].59161673E-03
[illegible].442[illegible]	[illegible]	[illegible].37732494E-03	[illegible].17204365E-03	0.60665743E-03
0.44908[illegible]67E 00	[illegible]E-04	0.39402040E-03	0.17158284E-03	0.62312216E-03
0.456[illegible]	[illegible]	[illegible].41[illegible]E-03	0.17106768E-03	[illegible].64160725E-03
[illegible].4629[illegible]	[illegible]	[illegible].43[illegible]9837E-03	[illegible].17048957E-03	[illegible].66157954E-03
[illegible].4698[illegible]	[illegible]	0.46132979E-03	0.16983780E-03	0.68[illegible]8432E-03

TABLE-5

[illegible] TION FOR Z = 40.000022 40.00000

[illegible]	ZG	ZP	ZH	ZD
0.1757[illegible]	[illegible]7760[illegible]-03	0.36276123E-04	0.16342492E-03	0.63177853E-03
0.1805994[illegible]	[illegible]	0.38046147E-04	0.16413245E-03	0.61573495E-03
[illegible]	[illegible]	0.39903118E-04	0.16477238E-03	0.59366081E-03
0.1914[illegible]	[illegible]	0.41552268E-04	0.16535196E-03	0.57799971E-03
0.1967[illegible]E 00	[illegible]	0.43399203E-04	0.16587883E-03	0.56210537E-03
0.2021[illegible]	[illegible]72659E-03	0.46049882E-04	0.16635834E-03	0.54813481E-03
0.2075[illegible]	[illegible]31797[illegible]E-03	0.48310628E-04	0.16679524E-03	0.53307740E-03
0.2113[illegible]	[illegible]	0.50688645E-04	0.16719357E-03	0.51968660E-03
0.2182[illegible]	0.28702978E-03	0.53191674E-04	0.16755678E-03	0.50777823E-03
0.22365519E 00	0.27333144E-03	0.55828258E-04	0.16788782E-03	0.49674752E-03
0.229[illegible]484[illegible]	0.26003758E-03	0.58607762E-04	0.16818924E-03	0.48683459E-03
0.2344315[illegible]	0.24705639E-03	0.61540481E-04	0.16846323E-03	0.47706011E-03
0.23981486E 00	0.23449866E-03	0.64613354E-04	0.16871171E-03	0.46782372E-03
0.24519209E 00	0.22384004E-03	0.67855775E-04	0.16893631E-03	0.45983213E-03
0.25058137E 00	0.21238558E-03	0.71280049E-04	0.16913849E-03	0.45280411E-03
0.25596454E 00	0.20176898E-03	0.74880783E-04	0.16931947E-03	0.44596923E-03
0.26134776E 00	0.19178734E-03	0.78663214E-04	0.16948033E-03	0.43993088E-03
0.26673099E 00	0.18256938E-03	0.82662462E-04	0.16962201E-03	0.43485385E-03
0.27211421E 00	0.17368354E-03	0.86853495E-04	0.16974529E-03	0.43028232E-03
0.27749744E 00	0.16523627E-03	0.91266815E-04	0.16985085E-03	0.42635393E-03
0.28288066E 00	0.15734581E-03	0.95919088E-04	0.16993926E-03	0.42320415E-03
0.28826389E 00	0.14967738E-03	0.10078212E-03	0.17001098E-03	0.42047048E-03
0.29364711E 00	0.14250065E-03	0.10592577E-03	0.17006639E-03	0.41849281E-03
0.29903034E 00	0.13560208E-03	0.11128571E-03	0.17010577E-03	0.41699356E-03
0.30441356E 00	0.12908550E-03	0.11694995E-03	0.17012931E-03	0.41616475E-03
0.30979678E 00	0.12284790E-03	0.12286369E-03	0.17013715E-03	0.41584874E-03
0.31518001E 00	0.11694285E-03	0.12910521E-03	0.17012931E-03	0.41617738E-03
0.32056323E 00	0.11128298E-03	0.13562292E-03	0.17010577E-03	0.41701166E-03
0.32594646E 00	0.10591430E-03	0.14252154E-03	0.17006639E-03	0.41850223E-03
0.33132968E 00	0.10077388E-03	0.14969273E-03	0.17001098E-03	0.42047673E-03
0.33671291E 00	0.95919628E-04	0.15736630E-03	0.16993925E-03	0.42322518E-03
0.34209613E 00	0.91256397E-04	0.16525898E-03	0.16985085E-03	0.42636822E-03
0.34747935E 00	0.86849575E-04	0.17372513E-03	0.16974528E-03	0.43031999E-03
0.35286258E 00	0.82657854E-04	0.18263265E-03	0.16962200E-03	0.43491249E-03
0.35824580E 00	0.78656644E-04	0.19186578E-03	0.16948032E-03	0.44000475E-03
0.36362903E 00	0.74878602E-04	0.20184654E-03	0.16931946E-03	0.44604460E-03
0.36901225E 00	0.71276028E-04	0.21240617E-03	0.16913848E-03	0.45282068E-03
0.37439548E 00	0.678[illegible]E-04	0.22314226E-03	0.16893630E-03	0.45993208E-03
0.37977870E 00	0.6462[illegible]E-04	0.23466753E-03	0.16871170E-03	0.46798976E-03
0.38516193E 00	0.615[illegible]E-04	0.24712642E-03	0.16846322E-03	0.47712811E-03
0.39054515E 00	0.586[illegible]E-04	0.26025[illegible]E-03	0.16818922E-03	0.48794617E-03
0.39592838E 00	0.558[illegible]E-04	0.27[illegible]E-03	0.16788781E-03	0.49702228E-03
0.40131260E 00	[illegible]E-04	0.28724786E-03	0.16755676E-03	0.50799568E-03
0.40669482E 00	[illegible]E-04	0.30225984E-03	0.1671935[illegible]E-03	0.52013978E-03
0.41207805E 00	[illegible]E-04	0.3185105[illegible]E-03	0.16679522E-03	0.53361354E-03
0.4174612[illegible]	[illegible]	0.3362828[illegible]E-03	0.1663583[illegible]	[illegible]54868845E-03
0.42284[illegible]	[illegible]	0.35560691E-03	0.16587881E-03	0.56[illegible]28411E-03
0.42822773E 00	[illegible]E-04	0.37123405E-03	0.16535193E-03	0.57840715E-03
0.4336109[illegible]	[illegible]	0.39037714E-03	0.16477206E-03	0.595[illegible]4452E-03
0.43899417[illegible]	[illegible]	0.41153869E-03	0.16413242E-03	0.61571645E-03
0.4443774[illegible]	0.3637[illegible]E-04	0.43886770E-03	0.16342487E-03	0.63476791E-03

TABLE-6.

[illegible] Z = [illegible].0000ZZ 50.00000

RT	ZS	ZP	ZH	ZD
0.[illegible]	[illegible]-03	0.30[illegible]62269E-04	0.15899673E-03	0.61711983E-03
[illegible]	[illegible]-03	[illegible].32716384E-04	[illegible].15969706E-03	[illegible].59626126E-03
[illegible].22622[illegible]	[illegible]-03	[illegible].34657884E-04	0.16033613E-03	[illegible].57930565E-03
[illegible].22065[illegible]	[illegible]-03	0.36538840E-04	0.16092044E-03	0.56426746E-03
0.2250[illegible]	[illegible]-03	0.38[illegible]15303E-04	0.16145547E-03	[illegible].54836694E-03
0.229[illegible]	[illegible]-03	[illegible].41[illegible]47165E-04	[illegible].16194586E-03	[illegible].53463122E-03
0.2338[illegible]	[illegible]-03	[illegible].43[illegible]90455E-04	[illegible].16239557E-03	[illegible].51985463E-03
[illegible].23827[illegible] 00	0.[illegible]-03	0.45851597E-04	0.16280802E-03	0.50632338E-03
0.2426[illegible]	[illegible]-03	0.48434431E-04	0.16318613E-03	[illegible].49465828E-03
[illegible].247[illegible]9119E	[illegible].26369265E-03	[illegible].51150184E-04	[illegible].16353248E-03	[illegible].48357531E-03
[illegible].2515[illegible]117E	[illegible].256[illegible]1904E-03	[illegible].54[illegible]06814E-04	[illegible].16384926E-03	[illegible].47387512E-03
0.2559[illegible]115E 00	[illegible].24317654E-03	0.57007153E-04	0.16413841E-03	0.46432240E-03
0.26032[illegible]	[illegible].23062973E-03	0.60162636E-04	[illegible].16440162E-03	[illegible].45519399E-03
[illegible].2647311[illegible]	[illegible].21918713E-03	[illegible].63481376E-04	[illegible].16464036E-03	[illegible].44730886E-03
[illegible].2691410[illegible]E	[illegible].20849599E-03	[illegible].66975654E-04	[illegible].16485592E-03	[illegible].44032756E-03
0.27355105E 00	0.19803166E-03	0.70655067E-04	0.16504942E-03	0.43373615E-03
0.2779610[illegible]E 00	0.18806096E-03	0.74519979E-04	0.16522184E-03	0.42780278E-03
0.282371[illegible]	[illegible].17886117E-03	[illegible].78572511E-04	0.16537403E-03	0.42280771E-03
0.28673[illegible]98E	[illegible].17[illegible]01592E-03	0.82842020E-04	0.16550671E-03	0.41836466E-03
0.29119095E 00	0.16159419E-03	0.87329839E-04	0.16562052E-03	0.41454455E-03
0.29560093E 00	[illegible].15369709E-03	[illegible].92021786E-04	0.16571597E-03	0.41143484E-03
0.30[illegible]0109[illegible]E 00	0.14606103E-03	0.96966582E-04	0.16579350E-03	0.40882111E-03
0.30442088E 00	0.13889992E-03	0.10214016E-03	0.16585344E-03	0.40689352E-03
0.30883085E 00	0.13199178E-03	0.10756410E-03	0.16589607E-03	0.40545195E-03
0.31324083E 00	0.12548133E-03	0.11326609E-03	0.16592157E-03	0.40466898E-03
0.31765080E 00	0.11929729E-03	0.11921437E-03	0.16593007E-03	0.40435173E-03
0.32206078E 00	0.11325137E-03	0.12548848E-03	0.16592157E-03	0.40466142E-03
0.32647076E 00	0.10755748E-03	0.13201102E-03	0.16589607E-03	0.40546457E-03
0.33088073E 00	0.10213281E-03	0.13892561E-03	0.16585344E-03	0.40691135E-03
0.33529070E 00	0.96958725E-04	0.14609138E-03	0.16579349E-03	0.40884359E-03
0.33970068E 00	0.92017994E-04	0.15372655E-03	0.16571596E-03	0.41146050E-03
0.34411065E 00	0.87324218E-04	0.16162617E-03	0.16562051E-03	0.41457089E-03
0.34852062E 00	0.82837077E-04	0.17005811E-03	0.16550670E-03	0.41840189E-03
0.35293059E 00	0.78571238E-04	0.17892446E-03	0.16537402E-03	0.42286972E-03
0.35734056E 00	0.74515619E-04	0.18813615E-03	0.16522183E-03	0.42787360E-03
0.36175053E 00	0.70651[illegible]E-04	0.19811066E-03	0.16504941E-03	0.43381190E-03
0.36616050E 00	0.66975222E-04	[illegible].20553346E-03	[illegible].16485590E-03	0.44036461E-03
0.37057047E 00	0.63479482E-04	[illegible].21925274E-03	0.16464034E-03	0.44737256E-03
0.37498044E 00	0.60159586E-04	0.23080963E-03	0.16440161E-03	0.45537082E-03
0.37939041E 00	0.57004803E-04	0.24329581E-03	0.16413839E-03	0.46443900E-03
0.38380038E 00	0.54[illegible]4966E-04	[illegible].25521399E-03	[illegible].16384924E-03	[illegible].47406819E-03
0.38821036E 00	0.51[illegible]E-04	0.26919879E-03	[illegible].16353246E-03	0.48388018E-03
0.39262033E 00	0.48[illegible]E-04	0.28[illegible]7014E-03	0.16318612E-03	0.49479026E-03
0.39703029E 00	[illegible]-04	0.29[illegible]E-03	[illegible].16280800E-03	[illegible].50691[illegible]59E-03
[illegible].40144027E 00	[illegible]-04	[illegible].31454666E-03	[illegible].16239555E-03	[illegible].52051251E-03
[illegible]	[illegible]-04	[illegible].33239572E-03	0.16194584E-03	[illegible]
[illegible].41026023E 00	[illegible]-04	0.34940806E-03	0.16145545E-03	0.5496[illegible]E-03
0.414670[illegible]E 00	[illegible]-04	0.36[illegible]93973E-03	[illegible].16092042E-03	[illegible].56454[illegible]
[illegible]	[illegible]-04	[illegible].38891891E-03	[illegible].16033611E-03	[illegible]
[illegible]	[illegible]-04	[illegible].40635591E-03	[illegible].15969702E-03	[illegible].59924079E-03
0.42790009E 00	[illegible]-03	0.42025966E-03	0.15899670E-03	0.620[illegible]E-03

TABLE.7

[illegible] RT [illegible] FOR Z1= [illegible].0000 Z2 60.00000

RT	ZS	ZP	ZH	ZD
0.2[illegible]	[illegible]-03	0.27[illegible]01108E-04	0.15555948E-03	0.60641871E-03
[illegible]	[illegible]	0.29[illegible]99583E-04	0.15624[illegible]23E-03	0.5853781[illegible]E-03
0.2569[illegible]	[illegible]-03	0.31[illegible]81815E-04	0.15686495E-03	0.56825[illegible]86E-03
0.24[illegible]65[illegible]	[illegible]-03	0.33457035E-04	0.15743908E-03	0.55381285E-03
0.2[illegible]	[illegible]-03	0.35[illegible]23992E-04	0.15796726E-03	0.53757541E-03
0.2[illegible]	[illegible]-03	0.37897637E-04	0.15845345E-03	0.52426272E-03
0.2518667[illegible]	[illegible]-03	0.40277150E-04	0.15890107E-03	0.5[illegible]965743E-03
0.2556066[illegible]E 00	0.[illegible]9[illegible]74970E-03	0.42773320E-04	0.15931307E-03	0.49583609E-03
0.2593424[illegible]	0.[illegible]79[illegible]3897E-03	0.45394079E-04	0.15969204E-03	0.48432509E-03
0.263[illegible]3[illegible]39E	[illegible].26491648E-03	0.48146804E-04	0.16004021E-03	0.47310349E-03
0.2668132[illegible]	[illegible].25215590E-03	0.51037519E-04	0.16035953E-03	0.46355295E-03
0.27055617E 00	0.23954632E-03	0.54073533E-04	0.16065176E-03	0.45427161E-03
0.27429407E 00	0.2270002[illegible]E-03	0.57262295E-04	0.16091837E-03	0.44518090E-03
0.278[illegible]3196E	[illegible].21558098E-03	0.60611571E-04	0.16116072E-03	0.43735327E-03
0.2817698[illegible]E	0.20487134E-03	0.64129968E-04	0.16137995E-03	0.43038176E-03
0.28550775E 00	0.19461361E-03	0.67827084E-04	0.16157708E-03	0.42401778E-03
0.28924564E 00	0.18468986E-03	0.71713741E-04	0.16175301E-03	0.41815661E-03
0.29298354E 00	0.17555245E-03	0.75799266E-04	0.16190850E-03	0.41326021E-03
0.29672143E 00	0.16680306E-03	0.80076797E-04	0.16204424E-03	0.40892409E-03
0.30045932E 00	0.15846115E-03	0.84562158E-04	0.16216078E-03	0.40518408E-03
0.30419721E 00	0.15062032E-03	0.89278354E-04	0.16225860E-03	0.40215728E-03
0.30793511E 00	0.14307221E-03	0.94216227E-04	0.16233812E-03	0.39962655E-03
0.31167300E 00	0.13599601E-03	0.99381787E-04	0.16239963E-03	0.39773743E-03
0.31541090E 00	0.12912825E-03	0.10481310E-03	0.16244341E-03	0.39638476E-03
0.31914879E 00	0.1226277[illegible]E-03	0.11047436E-03	0.16246960E-03	0.39557123E-03
0.32288668E 00	0.11639975E-03	0.11641451E-03	0.16247832E-03	0.39529258E-03
0.32662458E 00	0.110470[illegible]E-03	0.12263516E-03	0.16246960E-03	0.39557529E-03
0.33036247E 00	0.10480307E-03	0.12913485E-03	0.16244341E-03	0.39638133E-03
0.33410037E 00	0.9937778[illegible]E-04	0.13597805E-03	0.16239964E-03	0.39775544E-03
0.33783826E 00	0.942089[illegible]E-04	0.14310076E-03	0.16233812E-03	0.39964779E-03
0.34157615E 00	0.8927911[illegible]E-04	0.15065065E-03	0.16225860E-03	0.40218837E-03
0.34531404E 00	0.84587137E-04	0.15849705E-03	0.16216078E-03	0.40521496E-03
0.34905194E 00	0.80071[illegible]E-04	0.16684397E-03	0.16204424E-03	0.40895937E-03
0.35278983E 00	0.7579717[illegible]E-04	0.17561207E-03	0.16190851E-03	0.41331774E-03
0.35652773E 00	0.71713588E-04	0.18476050E-03	0.16175300E-03	0.41822709E-03
0.36026562E 00	0.67825749E-04	0.19469403E-03	0.16157708E-03	0.42409687E-03
0.36400351E 00	0.6412[illegible]E-04	0.20492652E-03	0.16137995E-03	0.43043461E-03
0.36774141E 00	0.60569724E-04	0.21560236E-03	0.16116072E-03	0.43737288E-03
0.37147930E 00	0.5726076[illegible]E-04	0.22[illegible]17589E-03	0.16091837E-03	0.44535502E-03
0.37521719E 00	0.54072470E-04	0.23971408E-03	0.16065176E-03	0.45443831E-03
0.37895508E 00	0.51[illegible]E-04	0.25224993E-03	0.16035953E-03	0.46364652E-03
0.38269298E 00	0.48[illegible]E-04	0.26525249E-03	0.16004019E-03	0.47343931E-03
0.38643087E 00	0.45398005E-04	0.27928120E-03	0.15969205E-03	0.48436653E-03
0.39016877E 00	0.42[illegible]-04	0.29[illegible]36898E-03	0.1593130[illegible]E-03	0.49645397E-03
0.39390666E 00	[illegible]	0.31[illegible]79773E-03	0.15890106E-03	0.50991497E-03
0.39764455E 00	[illegible]	0.33[illegible]787[illegible]E-03	0.15845344E-03	0.52[illegible]78[illegible]E-03
0.40138245E 00	[illegible]-04	0.34529215E-03	0.15796725E-03	[illegible]
0.40512034E 00	[illegible]-04	0.36[illegible]99966E-03	0.1574390[illegible]E-03	0.55389[illegible]
0.4[illegible]	[illegible]	0.38[illegible]92324E-03	0.15686494E-03	0.571686[illegible]E-03
[illegible]	[illegible]	0.40273868E-03	0.15624922E-03	0.5853782[illegible]E-03
0.41633402E 00	0.27800[illegible]82E-04	0.42337207E-03	0.1555594[illegible]E-03	0.60943213E-03

TABLE-8

[illegible] Z = [illegible] . [illegible] ZP 7.[illegible]

[illegible]	[illegible]	ZP	ZH	ZD
[illegible]	[illegible]	0.23336469E-04	0.15272403E-03	0.58424468E-03
[illegible]	[illegible]	0.25118646E-04	0.15338366E-03	0.56493010E-03
[illegible]	[illegible]	0.27182907E-04	0.15399169E-03	0.54914337E-03
0.2552349[illegible]	[illegible]	0.29135522E-04	0.15455224E-03	0.53392153E-03
[illegible]	[illegible]	0.31281177E-04	0.15506956E-03	0.51902687E-03
[illegible]	[illegible]	0.33525919E-04	0.15554713E-03	0.50548922E-03
[illegible]	[illegible]	0.35881028E-04	0.15598798E-03	0.49089280E-03
[illegible]	[illegible]	0.38344767E-04	0.15639475E-03	0.47823305E-03
[illegible]	[illegible]	0.40932886E-04	0.15676972E-03	0.46680013E-03
[illegible]	0.25576976E-03	0.43649498E-04	0.15711495E-03	0.45653420E-03
[illegible]	[illegible]	0.46498301E-04	0.15743216E-03	0.44712143E-03
[illegible]	0.23064499E-03	0.49491949E-04	0.15772295E-03	0.43785989E-03
[illegible]	[illegible]	0.52637570E-04	0.15798868E-03	0.42920689E-03
0.28768577E [illegible]	[illegible]	0.55942082E-04	0.15823055E-03	0.42172157E-03
[illegible]	0.19733717E-03	0.59414294E-04	0.15844964E-03	0.41520110E-03
0.29417594E 00	0.18711384E-03	0.63061023E-04	0.15864687E-03	0.40882173E-03
[illegible]	0.17769946E-03	0.66892352E-04	0.15882308E-03	0.40341489E-03
[illegible]	0.16879959E-03	0.70917809E-04	0.15897896E-03	0.39869636E-03
0.30391119E 00	0.16020905E-03	0.75142229E-04	0.15911514E-03	0.39446642E-03
0.30715628E 00	0.15218785E-03	0.79562712E-04	0.15923215E-03	0.39098271E-03
0.31040136E 00	0.14441639E-03	0.84202883E-04	0.15933043E-03	0.38794970E-03
0.31364644E 00	0.13711153E-03	0.89076889E-04	0.15941035E-03	0.38559877E-03
0.31689153E 00	0.13008560E-03	0.94169343E-04	0.15947221E-03	0.38372715E-03
0.32013661E 00	0.12343167E-03	0.99499042E-04	0.15951623E-03	0.38244715E-03
0.32338170E 00	0.11702198E-03	0.10509458E-03	0.15954258E-03	0.38165914E-03
0.32662678E 00	0.11091667E-03	0.11092678E-03	0.15955135E-03	0.38139501E-03
0.32987186E 00	0.10508620E-03	0.11703481E-03	0.15954256E-03	0.38166359E-03
0.33311694E 00	0.99494609E-04	0.12343463E-03	0.15951623E-03	0.38244555E-03
0.33636203E 00	0.94161287E-04	0.13010410E-03	0.15947221E-03	0.38373760E-03
0.33960712E 00	[illegible]	0.13713377E-03	0.15941034E-03	0.38561661E-03
0.34285220E 00	[illegible]	0.14443203E-03	0.15933043E-03	0.38796419E-03
0.34609728E 00	0.79558554E-04	0.15219137E-03	0.15923214E-03	0.39098206E-03
0.34934236E 00	[illegible]	0.16021116E-03	0.15911513E-03	0.39446445E-03
0.35258745E 00	[illegible]	0.16879741E-03	0.15897695E-03	0.39869045E-03
0.35583253E 00	0.66889121E-04	0.17770265E-03	0.15882307E-03	0.40341484E-03
0.35907762E 00	[illegible]	0.18713969E-03	0.15864686E-03	0.40884512E-03
0.36232270E 00	[illegible]	0.19146679E-03	0.15844962E-03	0.41526866E-03
0.36556778E 00	[illegible]	0.20766554E-03	0.15823053E-03	0.42183792E-03
0.36881287E 00	0.52635990E-04	0.21870006E-03	0.15798866E-03	0.42932470E-03
0.37205795E 00	[illegible]	0.23066839E-03	0.15772293E-03	0.43788138E-03
0.37530304E 00	[illegible]	0.24340929E-03	0.15743214E-03	0.44733899E-03
0.37854812E 00	[illegible]	0.25598275E-03	0.15711492E-03	0.45674605E-03
0.38179320E 00	[illegible]	0.26937305E-03	0.15676970E-03	0.46707455E-03
0.38503829E 00	[illegible]	[illegible]	0.15639471E-03	0.47857846E-03
[illegible]	[illegible]	0.29955672E-03	0.15598795E-03	[illegible]
[illegible]	[illegible]	0.31578127E-03	0.15554709E-03	[illegible]
[illegible]	[illegible]	[illegible]	0.15506952E-03	[illegible]
[illegible]	[illegible]	0.35882459E-03	[illegible]	[illegible]
[illegible]	[illegible]	0.36913276E-03	0.15399166E-03	[illegible]
[illegible]	[illegible]	0.38957759E-03	[illegible]	0.56787989E-03
[illegible]	[illegible]	0.41129664E-03	[illegible]	[illegible]

TABLE-9

[illegible] FOR ZI= 30.[illegible] Z2 20.00000

ST	ZG	ZP	ZH	ZD
[illegible]	0.49574579E-03	0.52689940E-04	0.17000978E-03	0.71844551E-03
[illegible]	[illegible]	0.54453730E-04	0.17104376E-03	0.69134783E-03
[illegible]	[illegible]	0.56307179E-04	0.17194879E-03	0.66824258E-03
[illegible]	0.41849501E-03	0.58263680E-04	0.17274729E-03	0.64970597E-03
[illegible]	[illegible]	0.60342631E-04	0.17345660E-03	0.63284529E-03
[illegible]	[illegible]	0.62514554E-04	0.17409043E-03	0.61580189E-03
[illegible]	[illegible]	0.64820842E-04	0.17465974E-03	0.60178544E-03
[illegible]	0.34844435E-03	0.67253467E-04	0.17517349E-03	0.58587130E-03
0.18746505E 00	0.32632479E-03	0.69811315E-04	0.17563904E-03	0.57177514E-03
0.19418879E 00	0.31075754E-03	0.72530702E-04	0.17606251E-03	0.55935074E-03
0.20091254E 00	0.29591719E-03	0.75366786E-04	0.17644904E-03	0.54773301E-03
0.20763628E 00	0.28237894E-03	0.78388783E-04	0.17680300E-03	0.53757071E-03
0.21436003E 00	0.26916423E-03	0.81544182E-04	0.17712813E-03	0.52783653E-03
0.22108377E 00	0.25607849E-03	0.84894814E-04	0.17742764E-03	0.51840144E-03
0.22780752E 00	0.24411618E-03	0.88404620E-04	0.17770435E-03	0.51022514E-03
0.23453126E 00	0.23285983E-03	0.92117669E-04	0.17796068E-03	0.50293817E-03
0.24125500E 00	0.22263551E-03	0.96010952E-04	0.17819879E-03	0.49684525E-03
0.24797875E 00	0.21227938E-03	0.10012543E-03	0.17842061E-03	0.49082542E-03
0.25470250E 00	0.20267378E-03	0.10443293E-03	0.17862782E-03	0.48573453E-03
0.26142624E 00	0.19392596E-03	0.10898997E-03	0.17882199E-03	0.48173792E-03
0.26814998E 00	0.18557075E-03	0.11374651E-03	0.17900448E-03	0.47832174E-03
0.27487371E 00	0.17749772E-03	0.11878961E-03	0.17917658E-03	0.47546391E-03
0.28159747E 00	0.17010577E-03	0.12403427E-03	0.17933945E-03	0.47347948E-03
0.28832120E 00	0.16389451E-03	0.12961065E-03	0.17949418E-03	0.47219933E-03
0.29504494E 00	0.15640084E-03	0.13539359E-03	0.17964177E-03	0.47143620E-03
0.30176868E 00	0.15019226E-03	0.14153582E-03	0.17978319E-03	0.47151127E-03
0.30849242E 00	0.14423035E-03	0.14795449E-03	0.17991935E-03	0.47210418E-03
0.31521617E 00	0.13868195E-03	0.15470611E-03	0.18005112E-03	0.47343918E-03
0.32193990E 00	0.13341341E-03	0.16190215E-03	0.18017935E-03	0.47549491E-03
0.32866364E 00	0.12841847E-03	0.16929516E-03	0.18030490E-03	0.47801853E-03
0.33538738E 00	0.12377948E-03	0.17725680E-03	0.18042860E-03	0.48144488E-03
0.34211112E 00	0.11933455E-03	0.18563718E-03	0.18055128E-03	0.48552300E-03
0.34883486E 00	0.11517360E-03	0.19424763E-03	0.18067382E-03	0.49009445E-03
0.35555860E 00	0.11125629E-03	0.20353134E-03	0.18079710E-03	0.49558472E-03
0.36228234E 00	0.10756769E-03	0.21358946E-03	0.18092206E-03	0.50207920E-03
0.36900698E 00	0.10412095E-03	0.22363592E-03	0.18104968E-03	0.50880654E-03
0.37572983E 00	0.10084244E-03	0.23427969E-03	0.18118104E-03	0.51630311E-03
0.38245356E 00	0.97796588E-04	0.24576894E-03	0.18131727E-03	0.52488279E-03
0.38917730E 00	0.94992389E-04	0.25809191E-03	0.18145967E-03	0.53448396E-03
0.39590104E 00	0.92246366E-04	0.27081102E-03	0.18160964E-03	0.54466502E-03
0.40262478E 00	0.89768497E-04	0.28362211E-03	0.18176882E-03	0.55515548E-03
0.40934852E 00	0.87400985E-04	0.29729584E-03	0.18193905E-03	0.56653665E-03
0.41607226E 00	0.85240464E-04	0.31195258E-03	0.18212243E-03	0.57931558E-03
0.42279600E 00	[illegible]	0.32772758E-03	0.18232154E-03	0.59326413E-03
[illegible]	[illegible]	0.34484968E-03	0.18253987E-03	0.60870755E-03
[illegible]	[illegible]	0.36205556E-03	0.18277957E-03	0.62442[illegible]
[illegible]	[illegible]	0.37913818E-03	0.18304665E-03	[illegible]
[illegible]	[illegible]	0.39770234E-03	0.18334623E-03	0.65754[illegible]
[illegible]	[illegible]	0.41822828E-03	0.18368548E-03	0.67706616E-03
[illegible]	[illegible]	0.43990777E-03	0.18407373E-03	0.69792[illegible]
[illegible]	0.72819871E-04	0.46432928E-03	0.18452844E-03	0.72167[illegible]

TABLE-10

[illegible] FOR Z1= 40.00000 Z2 20.00000

[illegible]	ZC	ZP	ZH	ZC
[illegible]	.49374379E-03	0.40455108E-04	0.16520501E-03	0.70140590E-03
[illegible]	.46521654E-03	0.42338392E-04	0.16646525E-03	0.67412017E-03
[illegible]	.4391[illegible]E-03	0.44312984E-04	0.16757927E-03	0.65100855E-03
[illegible]	.41761179E-03	0.46382193E-04	0.16857166E-03	0.63256564E-03
[illegible]	.397[illegible]954E-03	0.48552277E-04	0.16946148E-03	0.61544329E-03
[illegible]	0.37751643E-03	0.50830050E-04	0.17026388E-03	0.59861036E-03
[illegible]	.36[illegible]E-03	0.53222085E-04	0.17099104E-03	0.58448222E-03
0.22[illegible]	.341[illegible]4702E-03	0.55736749E-04	0.17165294E-03	0.56843670E-03
0.228[illegible]	.32397841E-03	0.58383035E-04	0.17225782E-03	0.55461926E-03
0.2337567[illegible] 00	0.30811214E-03	0.61162037E-04	0.17281261E-03	0.54208678E-03
0.2388[illegible]	.29335985E-03	0.64072283E-04	0.17332317E-03	0.53075530E-03
0.2440[illegible]	.27960262E-03	0.67139112E-04	0.17379450E-03	0.52053622E-03
0.249167[illegible]	.26622337E-03	0.70373289E-04	0.17423091E-03	0.51082756E-03
0.2543050[illegible] 00	0.25302541E-03	0.73744836E-04	0.17463616E-03	0.50140639E-03
0.259442[illegible]	.24113188E-03	0.77503489E-04	0.17501352E-03	0.49344888E-03
0.2645792[illegible] 00	0.22991813E-03	0.81043193E-04	0.17536592E-03	0.48632724E-03
0.26971633E 00	0.21946059E-03	0.84954170E-04	0.17569590E-03	0.48011066E-03
0.2748534[illegible]E 00	0.20919585E-03	0.89083334E-04	0.17600581E-03	0.47428499E-03
0.279990[illegible]E 00	0.19965796E-03	0.93384846E-04	0.17629773E-03	0.46934053E-03
0.28512758E 00	0.19094627E-03	0.97921541E-04	0.17657356E-03	0.46544136E-03
0.29026467E 00	0.18247055E-03	0.10266111E-03	0.17683506E-03	0.46196672E-03
0.29540175E 00	0.17455986E-03	0.10763828E-03	0.17708382E-03	0.45928196E-03
0.30053885E 00	0.16722968E-03	0.11284624E-03	0.17732137E-03	0.45739729E-03
0.30567592E 00	0.160[illegible]E-03	0.11830719E-03	0.17754911E-03	0.45598456E-03
0.31081301E 00	0.15366691E-03	0.12402290E-03	0.17776841E-03	0.45535822E-03
0.31595009E 00	0.14758398E-03	0.13001442E-03	0.17798055E-03	0.45537895E-03
0.32108717E 00	0.14147263E-03	0.13627711E-03	0.17818679E-03	0.45593653E-03
0.32622426E 00	0.13599603E-03	0.14280068E-03	0.17838837E-03	0.45726508E-03
0.33136134E 00	0.13076198E-03	0.14973637E-03	0.17858651E-03	0.45908486E-03
0.33649844E 00	0.12587279E-03	0.15705272E-03	0.17878245E-03	0.46170796E-03
0.34163551E 00	0.12124[illegible]E-03	0.16456410E-03	0.17897741E-03	0.46478162E-03
0.34677260E 00	0.11688265E-03	0.17257248E-03	0.17917271E-03	0.46862784E-03
0.35190968E 00	0.11283188E-03	0.18106189E-03	0.17936966E-03	0.47326342E-03
0.35704676E 00	0.10894271E-03	0.18974728E-03	0.17956968E-03	0.47825967E-03
0.36218385E 00	0.10534826E-03	0.19908611E-03	0.17977428E-03	0.48420865E-03
0.36732094E 00	0.10195327E-03	0.20917716E-03	0.17998508E-03	0.49111550E-03
0.37245801E 00	0.98743310E-04	0.21917346E-03	0.18020387E-03	0.49814064E-03
0.37759510E 00	0.958[illegible]E-04	0.22890477E-03	0.18043260E-03	0.50614748E-03
0.38273218E 00	0.92998479E-04	0.24[illegible]7910E-03	0.18067348E-03	0.51505105E-03
0.38786927E 00	0.904[illegible]E-04	0.25179363E-03	0.18092902E-03	0.52514178E-03
0.39300635E 00	0.87979470E-04	0.26[illegible]E-03	0.18120205E-03	0.53527685E-03
0.398[illegible]	.85[illegible]E-04	.27[illegible]E-03	.18149593E-03	.54615466E-03
0.40[illegible]	[illegible]E-04	.29267977E-03	.18181452E-03	.558[illegible]E-03
0.408[illegible] 00	.81676606E-04	0.30734296E-03	0.18216248E-03	0.57[illegible]E-03
[illegible] 00	.79[illegible]E-04	0.32319525E-03	0.18254541E-03	0.585[illegible]E-03
[illegible]	[illegible]E-04	0.34[illegible]7955E-03	0.18297[illegible]E-03	.6[illegible]E-03
[illegible]	.766[illegible]E-04	.35682786E-03	.18344[illegible]E-03	.61[illegible]E-03
[illegible]	.75[illegible]E-04	0.37385496E-03	0.18398121E-03	0.63[illegible]E-03
[illegible] 00	.74060870E-04	0.39222783E-03	0.18459208E-03	0.65[illegible]E-03
[illegible]	.729[illegible]E-04	.41251876E-03	.18529593E-03	.67[illegible]72469E-03
[illegible]	.71875[illegible]E-04	.43[illegible]06678E-03	.1861[illegible]E-03	.69[illegible]927E-03

TABLE-11

[illegible] FUNCTION FOR Z1= 5[illegible].0[illegible]000Z2 20.00000

RT	ZS	ZP	ZH	ZD
0.2198317[illegible]	[illegible].49[illegible]E-03	0.36[illegible]14921E-04	0.16254019E-03	0.6946[illegible]89E-03
0.2239933[illegible]	[illegible].46495[illegible]E-03	[illegible].36[illegible]01888E-04	0.16390262E-03	0.66715638E-03
0.2261544[illegible]	[illegible].438[illegible]E-03	0.4[illegible]379522E-04	0.16511159E-03	0.644[illegible]1443E-03
0.23231581E 00	[illegible].41[illegible]754[illegible]E-03	0.42554361E-04	0.16619272E-03	0.62562250E-03
0.23647718E 00	0.396[illegible]E-03	[illegible].44031447E-04	0.16716592E-03	0.6[illegible]833248E-03
0.2406385[illegible]	[illegible].376379[illegible]E-03	[illegible].47217076E-04	0.16804695E-03	0.59164382E-03
0.2447999[illegible]	[illegible].3[illegible]9591E-03	[illegible].49716616E-04	0.16884853E-03	0.57746105E-03
0.24896127E 00	0.339[illegible]E-03	0.52337530E-04	0.16958107E-03	0.56135192E-03
0.25312264E 00	0.32239849E-03	0.55[illegible]86032E-04	0.17025319E-03	0.54773770E-03
0.25728402E	[illegible].30633557E-03	[illegible].57971048E-04	0.17087214E-03	0.53517875E-03
0.26144537E 00	[illegible].29164257E-03	0.61000083E-04	0.17144406E-03	0.52408671E-03
0.26560674E 00	0.27774323E-03	0.64163781E-04	0.17197421E-03	0.51388121E-03
0.2697681[illegible]	[illegible].26425647E-03	[illegible].67485894E-04	0.17246715E-03	0.50420950E-03
0.27392947E 00	[illegible].25098860E-03	0.70976949E-04	0.17292684E-03	0.49489239E-03
0.27809083E 00	0.23904745E-03	0.74637692E-04	0.17335676E-03	0.48704190E-03
0.28225220E 00	0.22796066E-03	0.78460170E-04	0.17376002E-03	0.48018084E-03
0.28641357E 00	0.21729065E-03	0.82480109E-04	0.17413937E-03	0.47391012E-03
0.29057493E 00	0.20705042E-03	0.86686484E-04	0.17449730E-03	0.46823420E-03
0.29473630E 00	0.19766154E-03	0.91082626E-04	0.17483609E-03	0.46358025E-03
0.29889766E 00	0.18895069E-03	0.95707572E-04	0.17515780E-03	0.45981606E-03
0.30305903E 00	0.18038997E-03	0.10051004E-03	0.17546434E-03	0.45636435E-03
0.30722039E 00	0.1725719[illegible]E-03	0.10556261E-03	0.17575750E-03	0.45389200E-03
0.31138176E 00	0.1652[illegible]E-03	0.11082176E-03	0.17603894E-03	0.45210458E-03
0.31554312E 00	0.15817309E-03	0.11633862E-03	0.17631027E-03	0.45082191E-03
0.31970450E 00	0.1516[illegible]E-03	0.12209657E-03	0.17657301E-03	0.45032469E-03
0.32386585E 00	0.14547630E-03	0.12811691E-03	0.17682864E-03	0.45042185E-03
0.32802722E 00	0.13965[illegible]E-03	0.13440323E-03	0.17707860E-03	0.45113715E-03
0.33218858E 00	0.1341[illegible]E-03	0.14100833E-03	0.17732433E-03	0.45251615E-03
0.33634995E 00	0.12898790E-03	0.14787708E-03	0.17756728E-03	0.45443226E-03
0.34051131E 00	0.1242[illegible]E-03	0.15517640E-03	0.17780890E-03	0.45715575E-03
0.34467268E 00	0.11956738E-03	0.16266617E-03	0.17805069E-03	0.46028474E-03
0.34883405E 00	0.11526564E-03	0.17068624E-03	0.17829420E-03	0.46424608E-03
0.35299541E 00	0.11122424E-03	0.17905375E-03	0.17854109E-03	0.46881908E-03
0.35715678E 00	0.10742729E-03	0.18774045E-03	0.17879307E-03	0.47396081E-03
0.36131814E 00	0.10387855E-03	0.19710228E-03	0.17905204E-03	0.48003286E-03
0.36547951E 00	0.1005[illegible]E-03	0.20696470E-03	0.17932004E-03	0.48680145E-03
0.36964088E 00	0.974[illegible]7E-04	0.21692167E-03	0.17959932E-03	0.49392819E-03
0.37380224E 00	0.94[illegible]E-04	0.22761832E-03	0.17989236E-03	0.50198065E-03
0.37796361E 00	0.91741200E-04	0.23917277E-03	0.18020206E-03	0.51112605E-03
0.38212498E 00	0.892[illegible]E-04	0.25143628E-03	0.18053159E-03	0.52117762E-03
0.38628634E 00	0.8621[illegible]E-04	0.26[illegible]42458E-03	0.18088467E-03	0.53112847E-03
0.39044769E 00	0.846[illegible]E-04	0.27[illegible]466E-03	0.18126566E-03	0.54209768E-03
0.39460906E 00	0.82[illegible]E-04	0.28980360E-03	0.18167965E-03	0.55406565E-03
0.39877[illegible]	[illegible]E-04	0.30442871E-03	0.182132[illegible]E-[illegible]	[illegible].5672465[illegible]E-03
[illegible].40293179	[illegible].7[illegible]E-04	[illegible].32026817E-03	0.1826325[illegible]E-[illegible]	0.5818597[illegible]E-03
[illegible]	[illegible].77[illegible]	[illegible].33724079E-03	0.18[illegible]8791E-03	0.5978789[illegible]E-03
0.4112545[illegible]E 00	[illegible].75[illegible]E-04	0.35[illegible]13988E-03	0.18[illegible]039E-03	0.6128266[illegible]E-03
[illegible].4154158[illegible]	[illegible].74[illegible]	[illegible].37[illegible]177E-03	[illegible].18451446E-[illegible]	[illegible].6292694[illegible]
[illegible].41957726	[illegible].73[illegible]	[illegible].38921176E-03	[illegible].1853188[illegible]E-03	0.64688876E-03
[illegible].4237386[illegible]	[illegible].72[illegible]	[illegible].40104203E-03	0.1862486[illegible]E-03	0.6665435[illegible]E-03
0.42789998E 00	0.72[illegible]	0.43025895E-03	0.187[illegible]E-03	0.6888555[illegible]E-03

TABLE 12

DYNAMIC LOAD FACTORS BY BUCKINGHAM

EQUATION [30]

Design Load W	Dynamic Load W_d	Ratio W_d / W
625	1567	2.52
725	1465	2.02
2080	4580	2.2
2939	8326	2.84
4160	9770	2.35
5270	12195	2.31

4. CONCLUSIONS

4.1 The following are the main conclusions drawn out of the presen investigation :

a) The worst loading position of the gears is near the pitch point.

b) The maximum dynamic load factor is 2.86.

c) The dynamic load factor is different at different speeds. It attains the maximum value near the resonance frequence of the system and as the speed increases it decreases. At higher speeds the dynamic factor for all gear tooth combinations is approximate equal.

4.2 SCOPE FOR FURTHER WORK :

The present investigation has the following features in comparison with other work in this field :

(A) It is the first attempt to evaluate, through theoretical analysis, the combined mesh stiffness of the gear system. In earlier work, the mesh stiffness characteristics were either chosen arbitrarily or an estimate was made experimentally. Nakamura 23 chose a square wave pattern and Seireg 25 analysis with a trapezoidal pattern.

(B) In almost all the investigations, the transmission ratio was taken to be unity. In this case some of the characteristics are idential about the central positions. If the numbers of teeth of pinion and gear are different, this symmetry about the central positions is destroyed. In the

present investigation gear combinations with unequal number of teeth have also been analysed.

(C) However the present investigation is far from the ideal one. Some significant assumptions are made here. In order to refine the results obtained, this work should be extended in the following way :

(a) The damping factor should be determined accurately and the analysis should be done with the refined value of the damping factor.

(b) Some provisions should be made to take into account the frictional forces and the impact of the teeth.

(c) Though provisions have been made in the present analysis to take accounts of ~~the~~ effect of the pitch error and the profile error, no computation is performed with these errors. It is suggested that the dynamic factors should be computed in the presence of these errors.

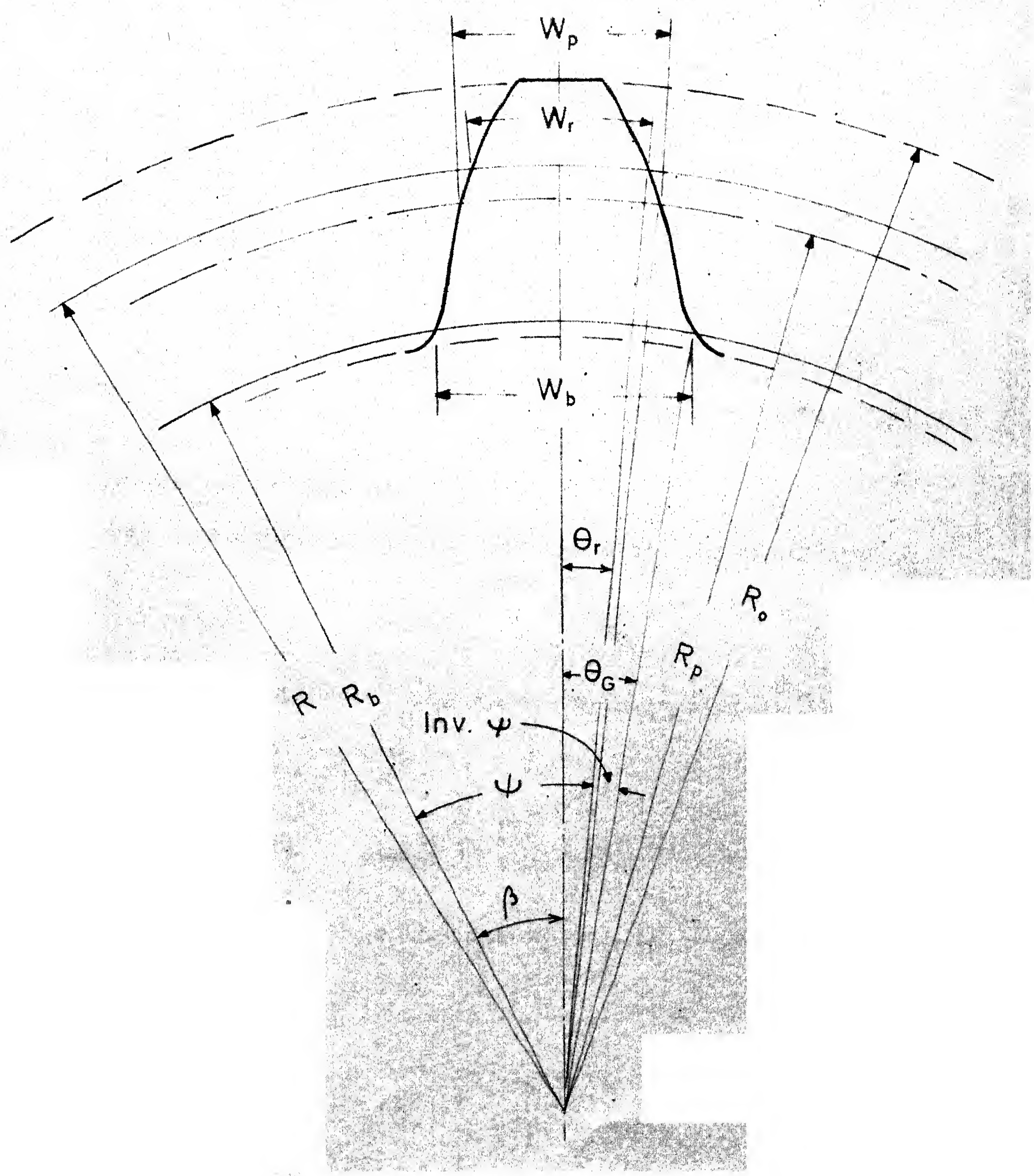


FIG. A-1. Tooth geometry

(b) THE INCLINATION OF THE NORMAL TO THE MEDIAN LINE

INCLINATION TO THE BASE-RADIUS

$$\beta = \psi + \text{inv}\ (\psi) - \theta_b$$

$$= \tan\psi - \theta_b$$

$$= \frac{\pi}{R_b} - \theta_b \qquad (4)$$

$$\beta' = \frac{\pi}{2} - \beta \qquad (5)$$

ρ = radius of curvature a of the profile at the position

(c) CO-ORDINATE TRANSFORMATION

The co-ordinates of the contact point in fixed system

$P\ (-R_b, \rho)$

The co-ordinates of the intersecting point on the median

$Q\ (-R_b, \ \rho - h)$

where $h = R_b\ (\tan\psi - \tan\beta)$

The transformation to the tooth-base-system is carried out according to,

$$\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = M_{or} \quad M_{rs} \begin{bmatrix} X_s \\ Y_s \\ 1 \end{bmatrix} \qquad (6)$$

$$M_{or} = \begin{bmatrix} 1 & 0 & -\bar{x} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (7)$$

$$M_{rs} = \begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (8)$$

Where

$$\bar{x} = R_o - \bar{x}_o$$

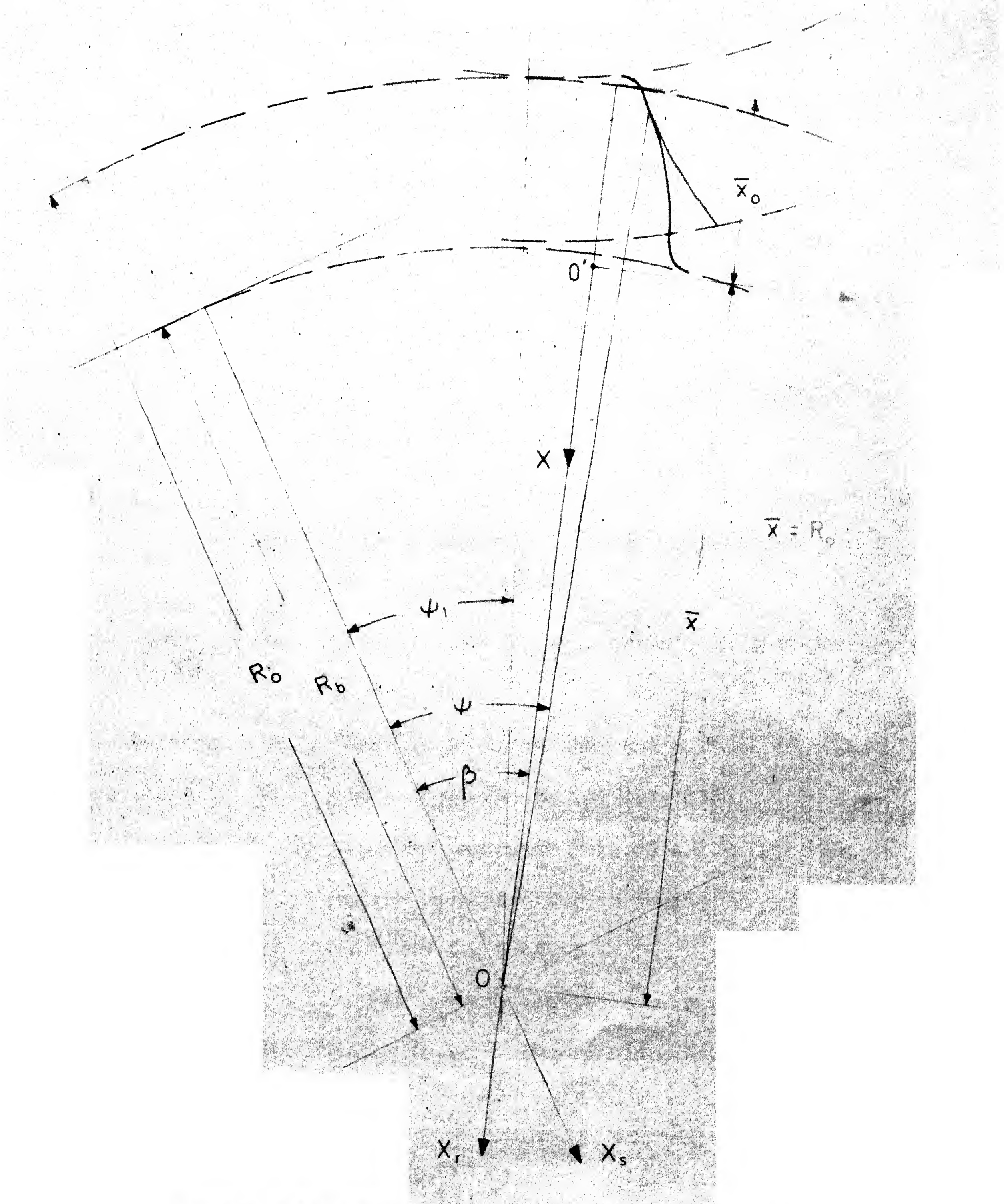


FIG. A-2. Co-ordinate transformation

REFERENCES

1. Max Cutler : "Strength of Gear-teeth", Trans. ASME, Vol. 37, 1915.

2. Ross, A.A. : "High speed-gears", Paper read at AGMA 1927.

3. Buchingham, E: "Dynamic Loads on Gear-teeth", ASME Research publication, Newyork, 1931.

4. Walker, H. : Gear-tooth deflections and profile modification, Engineer Oct., 1938.

5. Weber, C. : "The deformation of loaded gears and effect on their load carrying capacity", D.S.I.R. sponsored report No. 3 (Germany) - 1949.

6. Tuplin, W.A. : "Dynamic loads on gear-teeth", Machine-Design, Vol. 25, Oct. 1953.

7. Reswick, J.B. : "Dynamic Loads on spur and helical gear teeth", Trans. ASME, Vol. 77, No. 5, July 1955.

*8. Strauch, H. "Zahnrad-Schuingungen" Vol. 95 Z.V.D.I., 1953.

*9. Zeman, J. : "Dynamiche-zusatzkrafter in Zahnand getsneben", Vol. 79, Z.V.D.I., 1957.

*10. Richardson, H.H. : "Static and dynamic load stress and deflection cycles in spur gear-system", Sc.D. thesis, M.I.T., 1958.

11. Harris, S.L. : "Dynamic loads on the teeth of spur gears" Proc. Instn., Mech. Engr. London, 1958.

*12. Johnson, D.C. : "Excitation of resonant Vibrations by tooth meshing effects, Proc. Instn. Conf. Gearing, 1958.

*13. Neiman and Rellig, H. : "Error induced dynamic gear-tooth-load", Pro. Inst. Conf. Gearing 31, 1958, (Instn. Mech. Engrs., London).

*14. Weelauer, E.J., "An analysis of factors used for strength rating of Helical gears", ASME paper No. 59-A-121, 1959.

*15. Kohler, H. : "The Mechanism and measurement of dynamic loading in spur gearing", Ph.D. thesis, University of Sheffield, 1959.

16. Altia, A.Y.; "Dynamic loading of spur-gear-teeth", I.E.I., Trans.. ASME, series B, Vol. 81, No. 1, 1959.

*17. Woods, B. : "Sources of vibration excitation in spur-gear-teeth," Ph.D. thesis, University of Leads, 1960.

18. Utagawa, M. and Harada, T. :"Dynamic loads on spur gear teeth at high speed", Bull. J.S.M.E. 1961.

19. Aida, T. and Terauchi, Y. :"On bending of spur-gear teeth", Bull. J.S.M.E., Vol. 5, No. 17, 1962.

20. Ustinenko, V.L.; "Influence of some parameters on the bending stress in spur-gear teeth", (In Russian), Vestinik Mashinostroerya (Trans. of Machine-Design), No. 11, 1962.

21. Gregory, R.W., Harris, S.L. and Munro, R.G. :"Dynamic Behaviour of Spur-gears", Proc. Inst. Mech. Engrs., Vol. 178, No. 8, 1963-64.

*22. Bosch, M. Das dynamische Verhatten Von Stirnradgetrieben Untor besondorer Beracksichtigung der verzahnungsgenauigkcit", Industrie Anzeiger, Vol. 87, 1965.

23. Nakamura, K. : "Tooth Separation and abnormal noise on Power-transmission gears", Bull. JSME, Vol. 10, No. 4, 1967.

24. Neiman, G. : "Drehwegfehler, Zahnfenderharteund Gerausch bei Stirnradern", Z.V.D.I. Vol. 112, 1970.

25. Houser D.R. and Seireg. A. : "Evaluation of Dynamic factors for spur and Helical gears", J.E.I. Trans. A.S.M.E. Series B, Vol. 92, 1970.

26. Baronet C.N., and G.V. Toroion and A Premilhat : "An improved Detemination of elastic compliance of a spur gear-tooth acted on by a concentrated load", ASME paper No. 72, PTG.-9.

27. Ichimaru K, Hirano, F. :"Dynamic Behavior of heavy-loaded spur-gears", ASME paper No. 72- PTG-14.

28. Buckingham : "Analytical Mechanics of Gears", McGraw-Hill Book Co., N.Y.

29. Muskhelishvili, N.I. : "Some basic problems of the mathematical theory of elasticity", P. Noordhoffeto Greneingen Holland.

30. Dudley "Gear Hand Book, McGraw-Hill

31. ASME Special Committee Report "Dynamic Loads on Gears".(1931).

32. Mirrit : "Gears" I.P.S. Ltd.

33. M.M. Saverin - Increasing the Loading on Gearing and decreasing its weight

* Only cross reference because the articles are not available in Central Library, of I.I.T. Kanpur.

TAPERED GEARS

DEFLECTION

```
CIBJOB
CIBFTC MAIN
C     --------------------------------------------------------------
C
C            DEFLECTIONS OF GEAR  TOOTH
C
C     --------------------------------------------------------------
C       THIS  PROGRAMME  THE  STATIC  DEFLECTION  AND  STIFFNESS
C      BY  USING   THE  COMPLEX  VARIABLE  METHOD
      REAL INVP,INVT,INRO,NU ,INTV
      DIMENSION  BTX(200)
      DIMENSION  TABL(6,4)
        COMMON/LXX/ L
      COMMON/LY/NU
      DIMENSION T(6 ),BB(20,20),AA(20,20),BX(6 ),AX(6 ),ZA(5,10),
     1ZB(5,10),Z(5),RP(3),ROT(5),RB(5),RO(2,300),RIN(5),H(5),YB(5),XBR(
     25),ZG(300),ZP( 300),ZD(300), ZH(300)
      DIMENSION VG(200),UG(200),VP(200),UP(200),GXP(200),GYP(200),
     1GX(200),GY(200),PX(200),PY(200),PXP(200),PYP(200)   XS(10)  ,
     1 XBP(5),XBC(5)  ,A(4),B(4)
      DIMENSION  TABLE(200,4),TABGP(200,4),TABGC(200,4)
      DIMENSION  GXC(200),GYC(200)   ,PXC(200)(PYC(200)
      DIMENSION XX(2,200),XY(2,200),XZ(2,200)
      PRNG=20.
      PRINT102
      READ 100,(T(J),(AA(I,J),I=1,4),J=1,6)
      PRINT100,(T(J),(AA(I,J),I=1,4),J=1,6)
      PRINT101
      READ 100,(T(J),(BB(I,J),I=1,4),J=1,6)
      PRINT100,(T(J),(BB(I,J),I=1,4),J=1,6)
      PRINT104
      READ 100,Z(1),Z(2),XM,PRNG,W
      PRINT100,Z(1),Z(2),XM,PRNG,W
      PRNG=PRNG*3.1416/180.
      E=30E06
      XNU=.3
      G=E/(2.*(1.+XNU))
C
C      MAPPING  COEFF  CALCULATION
C
      DO  10  L=1,2
      TX=Z(L)
      DO  10  I=1,4
      DO  5  J=1,6
      BX(J)=BB(I,J)
      AX(J)=AA(I,J)
      CONTINUE
      CALL  DTABLE(T,BX,TABL ,6,2,TRUBL,4)
       IF(TRUBL.NE.0.0) GOTO  96
      ZB(L,I)=FNEWT(T,BX,TABL ,6,2,1,TX,TRUBL,4)
       IF(TRUBL.NE.0.0) GOTO  96
      CALL  DTABLE(T,AX,TABL ,6,2,TRUBL,4)
```

```
        IF(TRUBL.NE.1.0) GOTO  96
      ZA(L,I)=FNEWT(T,AX,TABL ,6,2,1,TX,TRUBL,4)
        IF(TRUBL.NE.0.0) GOTO  96
10    CONTINUE
C
C      TOOTH  PROFILE    CALCULATION
C
      DO  50  L=1,2
      DO  15  I=1,4
      A(I)=ZA(L,I)
      B(I)=ZB(L,I)
C     PRINT106,L,A(I),B(I)
15    CONTINUE
      GOTO(20,35),L
20    VG(1)=0.
      UG(1)=0.
C
C            PROFILE  FOR  GEAR
C
C     PRINT107,L
      DO  25  J=1,200
      V=VG(J)
      U=0.
      CALL  COMAP(U,V,A,B,X,Y)
      GXP(J)=X
      GYP(J)=Y
C     IF(J/20*20.EQ.J) PRINT108,V,X,Y
      VG(J+1)=VG(J)+.01
25    CONTINUE
C
C           CENTRAL  LINE  FOR  GEAR
C
C     PRINT109,L
      DO  30  J=1,200
      V=0.
      U=UG(J)
      CALL  COMAP(U,V,A,B,X,Y)
      GXC(J)=X
      GYC(J)=Y
C     IF(J/20*20.EQ.J) PRINT108,U,X,Y
      UG(J+1)=UG(J)+.005
30    CONTINUE
      GOTO  50
35    VP(1)=0.
      UP(1)=0.
C
C           PROFILE  FOR  PINION
C
C     PRINT107,L
      DO  [illegible]  J=1,[illegible]
      V=VP(J)
      U=0.
      CALL  COMAP(U,V,A,B,X,Y)
```

```
      PXP(J)=X
      PYP(J)=Y
C     IF(J/20*20.EQ.J) PRINT108,V,X,Y
      VP(J+1)=VP(J)+.01
40    CONTINUE
C
C         CENTRAL  LINE  FOR  PINION
C
C     PRINT109,L
      DO  45  J=1,200
      V=0.0
      U=UP(J)
      CALL  COMAP(U,V,A,B,X,Y)
      PXC(J)=X
      PYC(J)=Y
C     IF(J/20*20.EQ.J) PRINT108,U,X,Y
      UP(J+1)=UP(J)+.002
45    CONTINUE
50    CONTINUE
C
C          TOOTH  THICKNESS  AND  ORIGIN  LOCATION
C
      DO  65  L=1,2
      RP(L)=Z(L)*XM/2.
      ROT(L)=RP(L)+XM
      RIN(L)=RP(L)-1.25*XM
      RB(L)=RP(L)*COS(PRNG)
      SHIP=ARCOS(RB(L)/RP(L))
      SHIT=ARCOS(RB(L)/ROT(L))
      INVP=TAN(SHIP)-SHIP
      INVT=TAN(SHIT)-SHIT
      YCP=3.1416*XM/4.0
      YTIP=(YCP/RP(L)+INVP-INVT)*ROT(L)
      YTIP=YTIP/XM
      YB(L)=YCP/RP(L)+INVP
      GOTO(55,60),L
55    CONTINUE
      CALL  DTABLE(GYP,GXP,TABLE,200,3,TRUBL,4)
      XTIP=FNEWT(GYP,GXP,TABLE,200,3,2,YTIP,TRUBL,4)
      XTIP=XTIP*XM
      XBR(L)=ROT(L)-ABS(XTIP)
C     PRINT120,L,XTIP
      GOTO  65
 60    CALL  DTABLE(PYP,PXP,TABLE,200,3,TRUBL,4)
      XTIP=FNEWT(PYP,PXP,TABLE,200,3,2,YTIP,TRUBL,4)
      XTIP=XTIP*XM
      XBR(L)=ROT(L)-ABS(XTIP)
C     PRINT120,L,XTIP
65    CONTINUE
C
C       CALCULATION FOR  ENGAGEMENT  RANGE
C
      SUM=(RB(1)+RB(2))*TAN(PRNG)
```

```
      SH1=ARCOS(RB(1)/ROT(1))
      INRO=RB(1)*TAN(SH1)
      SH2=ARCOS(RB(2)/ROT(2))
      RO2=RB(2)*TAN(SH2)
      FNRO=SUM-RO2
      INTV=INRO-FNRO
      NSTEPS=100
      HSTEP=INTV/FLOAT(NSTEPS)
      RO(1,1)=INRO
      DO  70  I=2,101
70    RO(1,I)=RO(1,I-1)-HSTEP
      DO  75  I=1,101
75    RO(2,I)=SUM-RO(1,I)
C
C        DIVIDED  DIFF  TABLE FOR  GEAR
C
      CALL DTABLE(GXP,VG,TABGP,200,3,TRUBL,4)
        IF(TRUBL.NE.0.0) GOTO  96
      CALL DTABLE(GXC,UG,TABGC,200,3,TRUBL,4)
        IF(TRUBL.NE.0.0) GOTO  96
C      DIVIDED  DIFF  TABLE  FOR  PINION
      CALL DTABLE(PXP,VP,TABPP,200,3,TRUBL,4)
        IF(TRUBL.NE.0.0) GOTO  96
      CALL  DTABLE(PXC,UP,TABPC,200,3,TRUBL,4)
      IF(TRUBL.NE.0.0) GOTO  96
C          DEFLECTIONS  ALONG THE PATH OF V  CONTACT
      DO  95  J=1,101
      DO  90  L=1,2
      DO  76  K=1,4
      A(K)=ZA(L,K)
      B(K)=ZB(L,K)
76    CONTINUE
      XS(1)=-RB(L)
      XS(2)=RO(L,J)
      XS(3)=1.
      BT=RO(L,J)/RB(L)-YB(L)
      IF(L.EQ.2)BTX(J)=BT
      H(L)=RO(L,J)-RB(L)*TAN(BT)
      XBAR=XBR(L)
      CALL  TRSFM(XS,BT,XBAR,XBP)
      XP=XBP(1)/XM
      YP=XBP(2)/XM
      XS(2)=XS(2)-H(L)
      CALL  TRSFM(XS,BT,XBAR,XBC)
      XC=XBC(1)/XM
      YC=XBC(2)/XM
      GOTO(80,85),L
80    GALP=FNEWT(GXP,VG,TABGP,200,3,1,XP,TRUBL,4)
        IF(TRUBL.NE.0.0) GOTO  96
      GU=FNEWT(GXC,UG,TABGC,200,3,1,XC,TRUBL,4)
        IF(TRUBL.NE.0.0) GOTO  96
      BT=1.5708-BT
```

```
 130 FORMAT(10X,5F15.8)
      CALL  DFLN(W,GALP,GU,BT,A,B,ZW)
      ZG(J)=ZW
      GOTO  90
 85   PALP=FNEWT(PXP,VP,TABPP,200,3,1,XP,TRUBL,4)
        IF(TRUBL.NE.0.0) GOTO  96
      PU=FNEWT(PXC,UP,TABPC,200,3,1,XC,TRUBL,4)
        IF(TRUBL.NE.0.0) GOTO  96
      BT=1.5708-BT
      CALL  DFLN(W,PALP,PU,BT,A,B,ZW)
      ZP(J)=ZW
 90   CONTINUE
      HG=H(1)
      HP=H(2)
      XA=(1.-NU**2)/E
      XBB=(4.0*W*RO(1,J)*RO(2,J))/(3.1416*SUM)
      XCC=SQRT(2.*XA*XBB)
      XN=NU/(2.*(1.-NU))
      XZG=ALOG(2.*HG/XCC)-XN
      XZP=ALOG(2.*HP/XCC)-XN
      ZH(J)=2.*W*(1.-NU**2)*(XZG+XZP)/(3.1416*E)
      ZD(J)=ZG(J)+ZP(J)+ZH(J)
      PRINT160,BTX(J),ZG(J),ZP(J),ZH(J),ZD(J)
 95   CONTINUE
 96   PRINT115,TRUBL
      BTP=BTT(101)-BTT(1)
      DO  98  J=1,101
      BTX(J)=BTX(J)-BTP/2.-BTX(1)
      XX(J)=W/ZD(J)
 98   CONTINUE
C
C         INPUT  --OUTPUT  FORMATS
C
 100  FORMAT(5X,F10.5,4F15.8)
 101  FORMAT(5X,*NO-OF-TEETH*,*B1*,20X,*B2*,20X,*B3*,20X,*B4*,20X/)
 102  FORMAT(5X,*NO-ORD-TEETH*,*A1*,20X,*A2*,20X,*A3*,20X,*A4*,20X/)
 103  FORMAT(5X,2(F10.5,5X),3(F15.8,5X))
 104  FORMAT(5X,*GEAR TEETH*,6X,*PINION TEETH*,6X,*MODULE*,10X,*PR
     1 ANGLE*,12X,*LOAD*/)
 105  FORMAT(5X,*MAPPING COEFF FOR  GEAR-*,I2/5X,20(1H-)/5X,*L*,10X,*A1*,15X,*
     1 ,15X,*B1*, /)
 106  FORMAT(5X,I2,5X,F15.8,5X,F15.8/)
 107  FORMAT(5X,*PROFILE FOR GEAR-*,I2/5X,15(1H-)/10X,*V*,20X,*X*,20X,*Y*/)
     1*,15X/)
 108  FORMAT(5X,3(F15.8,5X))
 109  FORMAT(5X,*CENTRAL  LINE OF  GEAR -*,I2/5X,20(1H-)/
     1 10X,*U*,20X,*X*,20X,*Y*/)
 110  FORMAT(5X,I4,F15.8)
 115  FORMAT(10X,*TRUBL*,F .2)
 120  FORMAT(5X,*TOOTH TIP  CO-ORD FOR GEAR*,I2,*IS*,F15.8/)
 125  FORMAT(/5X,*FOR  GEAR *,I2,5X/5X,*LOAD  POSITION=*,F15.8/5X,*INTERSECTI
     1SECTION  WITH  C L-*,F15.8/5X,*RCSLINATION*,F15.8/5X,30(1H-)/)
 160  FORMAT(5X,5E15.8)
```

```
      STOP
      END
CIBFTC COMP
      SUBROUTINECOMAP(U,V,A,B,X,Y)
      DIMENSION  A(5),B(5)
      COMPLEX  Z,ZHI,SUM,CMPLX
  5   ZHI=CMPLX(U,V)
      SUM=(0.,0.0)
      DO  10  I=1,4
 10   SUM=SUM+A(I)/(ZHI+B(I))
      Z=ZHI-SUM
      X=REAL(Z)
      Y=AIMAG(Z)
      RETURN
      END
C
CIBFTC TRSF
      SUBROUTINE  TRSFM(XS,BT,XBAR,X)
C     THE  CO-ORD. OF THE CONTACT POINT  IN THE  TOOTH  SYSTEM OF
C     CO-ORD. ARE CALCULATED BY USING TRANSFORMATION  MATRICES
      DIMENSION  A(10,10),B(10,10),C(10,10),XS(10),X(10)
      N=2
      NN=N+1
C
C           A-MATRIX  INITIALIZATION
C
      A(1,1)=COS(BT)
      A(1,2)=-SIN(BT)
      A(1,3)=0.
      A(2,1)=SIN(BT)
      A(2,2)=COS(BT)
      A(2,3)=0.0
      A(3,1)=0.0
      A(3,2)=0.0
      A(3,3)=1.
C
C         B--MATRIX -INITIALIZATION
C
      DO  10  I=1,NN
      DO  10  J=1,NN
      B(I,J)=0.0
      IF(I.EQ.J)  B(I,J)=1.
 10   CONTINUE
      B(1,3)=XBAR
C
C         MATRIX  --MULTIPLICATION
C
      DO  20  I=1,NN
      DO  20  J=1,NN
      C(I,J)=0.0
      DO  20  K=1,NN
      C(I,J)=C(I,J)+B(I,K)*A(K,J)
 20   CONTINUE
```

```
C
C      TOOTH--SYSTEM --CO-ORDINATES
C
      DO  30  I=1,NN
      X(I)=0.
      DO  30  J=1,NN
      X(I)=X(I)+C(I,J)*XS(J)
30    CONTINUE
C     PRINT90,L,J
C     PRINT100,(XS(I),I=1,2),(X(I),I=1,2)
 90   FORMAT(5X,*CO-ORD  TRANS  FOR  GEAR*,I2,*FOR POSITION*,I4/)
100   FORMAT(5X,2F15.8/5X,2F15.8/)
      RETURN
      END
CIBFTC DTAB
      SUBROUTINE  DTABLE(X,Y,TABLE,N,M,TRUBL,K)
C     THE  DIVIDED DIFF.  TABLE  FOR  THE  INPUT VALUES OF X,Y,
     ARE CALCULATED FOR  THE  REQUIRED ORDER AND RETURNED TO THE
C     CALLING PROGRAMME FOR THE  FURTHER INTERPOLATION
      DIMENSION  X(N),Y(N),TABLE(N,K)
      IF(M.LT.N)GOTO  2
      TRUBL=1.
      RETURN
C       CALCULATE  FIRST ORDER DIFF
2     NM1=N-1
      DO  3  I=1,NM1
3     TABLE(I,1)=(Y(I+1)-Y(I))/(X(I+1)-X(I))
      IF(M.LE.1)GOTO  6
C          CALCULATE HIGHER-ORDER DIFF
      DO  5  J=2,M
      DO  5  I=J,NM1
      ISUB=I+1-J
5     TABLE(I,J)=(TABLE(I,J-1)-TABLE(I-1,J-1))/(X(I+1)-X(ISUB))
6     TRUBL=0.0
      RETURN
      END
CIBFTC FNEW
      FUNCTION  FNEWT(X,Y,TABLE,N,M,IDEG,XARG,TRUBL,K)
      DIMENSION  X(N),Y(N),TABLE(N,K)
      IF(IDEG.LE.M) GOTO  2
      TRUBL=1.
      FNEWT=0.0
      RETURN
C        SEARCH  X-VECTOR FOR ELEMENT.75=7 97
2     DO  4  I=1,N
4     IF(I.EQ.N.OR.XARG.LE.X(I)) GOTO    5
5     MAX=I+IDEG/2
C            INSURE  THAT ALL  REQUIRED-DIFF  ARE IN TABLE
      IF(MAX.GT.N) MAX=N
      IF(MAX.LE.IDEG) MAX=IDEG+1
C       COMPUTE  INTERPOLANT-VALUE
      YEST=TABLE(MAX-1,IDEG)
```

```
      IF(IDEG.LE.1) GOTO  13
      IDEGM1=IDEG-1
      DO  12  I=1,IDEGM1
      ISUB1=MAX-1
      ISUB2=IDEG-1
12    YEST=YEST*()ARG-X(ISUB1))+TABLE(ISUB1-1,ISUB2)
13    ISUB1=MAX-IDEG
      TRUBL=0.0
      FNEWT=YEST*(XARG-X(ISUB1))+Y(ISUB1)
      RETURN
      END

CIBFTC DFLN
      SUBROUTINE     DFLN(WW,ALP,U,BT,C,D,DP)
C     COMPLEX  VARIABLE-METHOD  FOR  TOOTH DEFLECTION
      DIMENSION  C(5),D(5)
      COMPLEX  A(5),B(5)
      COMPLEX  DPXY,DW,LW,MW   ,ZG,ZXM
      COMPLEX  ZTW,ZTX,Z,CBT,ZZ,WX,DWX,ZA,W,DPHI,ZPHD,PHIZ,SHIZ,CDW,WNX,BDW(Z
     1BDW(5),ZBPD(5),CEXP,CMPLX,CLOG,ZB,ZC
      COMMON/LA/ZZ,BDW
      DO  5  I=1,4
      A(I)=CMPLX(C(I),0.0)
      B(I)=CMPLX(D(I),0.0)
5     CONTINUE
      XMU=.3
      E=30E06
      G=E/(2.*(1.+XMU))
C     PRINT100
      LW=CMPLX(WW,0.0)
      ZTW=CMPLX(0.,ALP)
      ZTX=CMPLX(U,0.)
C     PRINT101,LW,ZTW,ZTX
101   FORMAT(5X,6F15.8)
C     PRINT107
107   FORMAT(5X,*DERVATIVE  AT  B-I*)
      DO  10  I=1,4
      ZB=CMPLX(B(I),0.)
      CALL  FUND(ZB,A,B,DW)
C     PRINT108,DW
108   FORMAT(10X,2F15.8)
10    BDW(I)=DW
      CBT=CMPLX(0.,BT)
      ZZ=   CEXP(CBT)/(6.2836,0.)
      CALL    SOLN(ZTW,ZBPD,A,B,ALP)
      ZA=ZTX-ZTW
      CALL  FUN(ZTX,A,B,WX)
      CALLFUND(ZTX,A,B,DWX)
      CALL  FUN(-ZTX,A,B,WNX)
C     PRINT105
105   FORMAT(5X,*TRANSFORM  FUN  VALUES*/)
C     PRINT104,ZA,WX,DWX,WNX
      PHIZ=-ZZ*CLOG(ZA)
```

```
      ZPHD=-ZZ/ZA
      DO  20  I=1,4
      CDW =CONJG(ZBPD(I))/CONJG(BDW(I))
      PHIZ=PHIZ+A(I)*CDW/(ZTX+B(I))
      ZPHD=ZPHD-A(I)*CDW/((ZTX+B(I))**2)
20    CONTINUE
      ZZ=   CEXP(-CBT)/(6.2832,0.0)
      SHIZ=ZZ*CLOG(ZA)-WNX*ZPHD/DWX

      DO  30  I=1,4
      SHIZ=SHIZ-A(I)*ZBPD(I)/((B(I)-ZTX)*BDW(I))
30    CONTINUE
      XMU=3.-4.*XNU
      ZXM=CMPLX(XMU,0.0)
110   FORMAT(15X,*MATERIAL  CONSTS G 'NU*/20X,2E15.8)
      GR=.5/G
      ZG=CMPLX(GR,0.0)
      DPXY=(ZXM*PHIZ-WX*CONJG(ZPHD)/CONJG(DWX)-CONJG(SHIZ))*ZG
C     PRINT106
106   FORMAT(5X,*STRESS  FUN  VALUES*/)
C     PRINT104,PHIZ,ZPHD,SHIZ,DPXY
104   FORMAT(5X,8E15.8/)
      DPXY=DPXY*LW
      DPX=REAL(DPXY)
      DPY=AIMAG(DPXY)
      DP=CABS(DPXY)
C      ---INPUT-OUTPUT FORMATS
100   FORMAT(10X,*LOAD  AND ITS  POSITION*/)
102   FORMAT(10X,*DEFLECTIONS*,I2/)
103   FORMAT(5X,3E15.8)
      RETURN
      END
CIBFTC  FUN
       SUBROUTINE  FUN(ZT,A,B,W)
      COMPLEX  W
      COMPLEX  A(5),B(5)
      COMPLEX  ZZ,BDW(5)
      COMMON/LA/ZZ,BDW
      COMPLEX  ZT
      W=ZT
      DO  10  I=1,4
      W=W-A(I)/(ZT+B(I))
10    CONTINUE
      RETURN
      END
CIBFTC  FUND
      SUBROUTINE  FUND(ZT,A,B,DW)
      COMPLEX   DW
      COMPLEX  A(5),B(5)
      COMPLEX  ZZ,BDW(5)
      COMMON/LA/ZZ,BDW
      COMPLEX  ZT
      DW=(1.,.0)
```

```
      DO  10  I=1,4
      DW=DW+A(I)/((ZT+B(I))**2)
 10    CONTINUE
      RETURN
      END

CIBFTC SOLN
      SUBROUTINE  SOLN(ZTW,ZBPD,AA,BB,ALP)
      COMPLEX  AA(5),BB(5)
      DIMENSION  A(5),B(5)
      COMPLEX ZTW
      COMPLEX  BDW(5),ZX(5),ZBPD(5),ZC,ZZ,CMPLX
      DIMENSION  D(10,10),XA(10,10),BX(4,4),BY(4,4),CX(4),CY(4)
      DIMENSION  CZ(4,1),CW(4,1)
      COMMON/LA/ZZ,BDW
      DO  5  I=1,4
      A(I)=REAL(AA(I))
      B(I)=REAL(BB(I))
 5    CONTINUE
      DO  10  I=1,4
      DO  10  J=1,4
      D(I,J)=0.
      IF(I.EQ.J) D(I,J)=1.
 10   CONTINUE
      DO  20  I=1,4
      ZC=CMPLX(B(I),-ALP)
      ZX(I)=-ZZ/ZC
      CX(I)=REAL(ZX(I) )
      CY(I)= AIMAG(ZX(I))
      CZ(I,1)=CX(I)
      CW(I,1)=CY(I)
      DO  20  J=1,4
      WB=REAL(BDW(J))
      XA(I,J)=A(J)/(B(I)+B(J))**2
      BX(I,J)=XA(I,J)/WB+D(I,J)
      BY(I,J)=XA(I,J)/WB-D(I,J)
 20   CONTINUE
      CALL  MATINV(BX,4,CZ,1,DETERM)
      CALL  MATINV(BY,4,CW,1,DETERM)
      DO  40  I=1,4
      CX(I)=CZ(I,1)
      CY(I)=CW(I,1)
      ZBPD(I)=CMPLX(CX(I),-CY(I))
 40   CONTINUE
      RETURN
      END
CFETCH MATINV  CCS999
CDONE
CENTRY
```

```
C       THE LISTING OF THE PROGRAMME  FOR  SOLVING
C       THE SECOND ORDER  DIFF  EQN OF  THE VIBRATIONAL  MODEL
C       ---------------------------------------------------------

CIBFTC MAIN
C
C        THE  DYNAMIC  LOADS  ON  GEARS
C
C     THE PROGRAMME  READS  THE  DATA  FOR  THE GEAR  SET UNDER CONSIDERATION
C       CALLS THE  STIFF  SUBROUTINE  FOR  CALCULATING THE MESS STIFFNESS
C       AND  ERROR IN ACTION  AND THE STATIC  DISPLACEMENT
C       FROM THE STATIC  DATA  THE  DIFF .EQN.  IS  SOLVED  USING THE  CLASSICAL
C       RUNGE -KUTTA  METHOD    THE  COEFF ARE CALCULATED  BY CALLING ON
C       THE SUBFUNCTION RUNGE .  FORM THE DYNAMIC DISPLACEMENTS  THE
C       DYN. LOADS ARE CALCULATED BY  USING  THE  MESS STIFNESS DTA AND THE
C       ERROR IN  ACTION.    THE MAX LOAD ON EACH  TOOTH-PAIR IS CALCULATED
C       THE  EQN. IS  SOLVED FOR  ARANGE OF  SPEEDS
        INTEGER  RUNGE
        DIMENSION  XX(200),XY(200) ,XZ(250),XS(300),P1(300),P2(300),PT(300)
       1),RN(50)
        DIMENSION  X(2),F(2)
        DIMENSION XK(250),XK1(250),XK2(250),E1(250),E2(250),QJ(250)  ,Z(5)
        COMMON/LA/E,PHI,DP,G
        COMMON/LYY/UMG,UMX,UMC
        COMMON/XYL/QZ,QT
C          STATIC  DEFLECTION
        READ80,Z(1),Z(2),DP
        PRINT80,Z(1),Z(2),DP
        PHI=.349
        PS=1000.
        ZHI=.1
        JJ=50
        NN=2*JJ+1
        PER=0.0
        CALL   STIFF(Z,XK,XK1,XK2,E1,E2,NN,XS,PS,PER)
C        INITIALISE THE VALUES
        EPS=.00001
        XOI=1.5
        XOJ=0.
        R=Z(1)/(2.*DP)
        PRANG=PHI
        XMSS=10.
        EQM=XMSS/2.
        CR=QT/QZ
        UMC=SQRT(EQM/AKT)
        RN(1)=0.
        JN=JJ+1
        PRINT88
```

```
 88     FORMAT(10X,*ENGAGEMENT DATA EQ  MASS  CRITICAL  SPEED*/)
        PRINT90,QT,QZ,CR,EQM,UMC
        DO  30  JK=2,10
        RN(JK)=RN(JK-1)+500.
        UMG=6.2832*RN(JK)/60.
        TMAX=60./(RN(JK)*Z(1))
        H=TMAX/FLOAT(JJ)
        PRINT89
 89     FORMAT(10X,*THE INITIAL  VALUES*/10X,15(1H-))
        PRINT90,XOI,XOJ,H,TMAX,UMG
        ICOUNT=0
        EPS=.1E-5
 5       T=0.
        J=1
        N=1
        X(1)=XOI
        X(2)=XOJ
        ICOUNT=ICOUNT+1
        PRINT95,ICOUNT,X(1),X(2)
 C       CALL THE RUNGE KUTTA  FUNCTION
 11     CONTINUE
        K=RUNGE(2,X,F,T,H,N)
        IF(K.NE.1)GOTO  13
        F(1)=X(2)
        UMSG=XK(N)/EQM
        UMSQT=SQRT(UMSQ)
        F(2)=-UMSQ*X(1)-2.*UMSQT*ZHI*X(2)
        GOTO  11
 13     CONTINUE
        XX(J)=T*UMG
        XY(J)=1.+X(1)
        XZ(J)=X(2)
        IF(J.GE.JN)GOTO  16
        J=J+1
        GOTO  11
 16     CONTINUE
        X1=ABS((X(1)-XOI)/XOI)
        X2=ABS((X(2)-XOJ)/XOJ)
        IF(X1.LE.EPS.AND.X2.LE.EPS)  GOTO  25
        XOI=(XOI+X(1))/2.
        XOJ=(XOJ+X(2))/2.
        GOTO  5
 25     PRINT96
 96     FORMAT(5X,*THE FINAL VALUES*/5X,*THETA*,10X,*DISPLACEMENT*,2X,*
       1VELOCITY*/)
        DO  26  M=1,JN
        MM=2*M-1
        XY(M)=XY(M)*XS(MM)
        P1(M)=XK1(MM)*(XY(M)-E1(MM))
        P2(M)=XK2(MM)*(XY(M)-E2(MM))
        PT(M)=P1(M)+P2(M)
 C      PUNCH100,XX(M),XY(M)
        PRINT100,XX(M),XY(M),XZ(M),P1(M),P2(M),PT(M)
```

```
 [illegible]  FORMAT(10X,6E15.8)
 26     CONTINUE
        PMAX1=P1(1)
        MAXI=1
        DO  27  M=1,JN
        IF(P1(M).LE.PMAX1)GOTO  27
        PMAX1=P1(M)
        MAXI=M
 27     CONTINUE
        PMAX2=P2(1)
        MAX2=1
        DO  28  M=1,JN
        IF(P2(M).LE.PMAX2)GOTO  28
        PMAX2=P2(M)
        MAX2=M
   28   CONTINUE
        PRINT120,RN(JK),PMAX1,MAXI,PMAX2,MAX2
 120    FORMAT(10X,*SPEED IN  RPM=*,F15.8/10X,*MAX LOAD ON TOOTH--1=*,
       1F15.8/10X,*AT POSITION J=*,I4/10X,*MAX LOAD ON TOOTH --2 IN LB=*,
       2F15.8/10X,*AT  POSION  J=*,I4/10X,30(1H-))
 30       CONTINUE
  90    FORMAT(10X,9F7.2)
 95     FORMAT(5X,I2,2E15.8)
        STOP
        END
 [illegible]
        INTEGER  FUNCTION  RUNGE(N,Y,F,X,H,L)
 C      [illegible] ROUTINE CALCULATES THE  COEFF  OF THE  RUNGE KUTTA  EQN.
 C      WHENEVER THIS FUNCTION IS CALLED FROM THE  MAIN  PROGRAMME
        DIMENSION  PHI(50),SAVEY(50),  Y(N),F(N)
        DATA  M/0/
        M=M+1
        GOTO(1,2,3,4,5),M
 C             PASS--1
 1       RUNGE=1
         RETURN
 C              PASS--2
 2       DO  22  J=1,N
        SAVEY(J)=Y(J)
        PHI(J)=F(J)
 22     Y(J)=SAVEY(J)+.5*H*F(J)
        X=X+.5*H
        L=L+1
        RUNGE=1
        RETURN
 C             PASS----3
 3      [illegible]     J=1,N
        PHI(J)=PHI(J)+2.0*F(J)
 [illegible] Y(J)=SAVEY(J)+.5*H*F(J)
        [illegible]
        [illegible]
 C                  PASS----4
        [illegible]     J=1,N
```

```
      PHI(J)=PHI(J)+2.0*F(J)
44    Y(J)=SAVEY(J)+H*F(J)
      X=X+.5*H
      L=L+1
      RUNGE=1
      RETURN
C               PASS---5
5      DO  55  J=1,N
55    Y(J)=SAVEY(J)+(PHI(J)+F(J))*H/6.0
      M=0.
      RUNGE=0
      RETURN
      END
CIBFTC STIFF
      SUBROUTINE  STIFF(Z,XK,XK1,XK2,E1,E2,NN,X,PS,EP)
C     THE  ROUTINE  READS  THE STATIC DEFLECTIONDATA- STIFNESS -FFTTHETWO6
C     TEETH IN  CONTACT  AS CALU
C     TEETH  IN  CONTACT . AS  CALCULATED  BY THE  DEFLECTION  PROGRAMME
C     THEN  THE  MESS-STIFNESS, ERRORIN ACTION AND THE STATIC DEFLECTION
C     ARE  CALCULATED  FOR  THE  INTERVAL OF ONE BASE PITCH THE STATIC
C     LOAD DISTRIBUTION IS ALSO  CALCULATED
C     IN FINDING THE MESS-STIFNESS FOR THE SET  THE INCREASING IN CONTACT RATI
C     IS  CALCULATED BY  FINDING  THE ADVANCE AND THE POSTPHONEMENT IN
C     THEENGAGEMENT AND THE DISENGAGEMENT POSITIONS  THE IS ACCOMPLISHED  BY C
C     LING  OF THE  SUBROUTINE  ADVN
      DIMENSION XK(250),XK1(250),XK2(250),E1(250),E2(250),QJ(250)  ,Z(5)
      COMMON/XYL/QZ,QT
      COMMON/LYY/UNG,UMX,UMC
      COMMON /YL/TABLE(101,4)
      COMMON/LZZ/BTL,BTU
      COMMON/XL/BTT(101),XS(101)
      COMMON/LA/G,PHI,DP,C
      DIMENSION  EROR(25),ADNG(25)  ,X(300)
      DIMENSION  P1(250),P2(285)  ,PT(250)
      DO  2  I=I,101
      READ110,BTT(I),XS(I)
110   FORMAT(10X,4E15.8)
2     CONTINUE
      BTP=BTT(101)-BTT(1)
      QZ=6.2832/Z(2)
      BTP=BTP*6.2832/QZ
      QT=BTP
      BTU=BTP/2.
      BTL=QZ-BTU
      DO [illegible]  I=[illegible]
[illegible] PRINT[illegible],BTT(I),XS(I)
      DO [illegible]  I=[illegible]
      BTT(I)=6.[illegible]*BTT(I)/QZ
      CALL DTABLE([illegible],TRUBL)
      N=NN-[illegible]
      HQ=6.2832/FLOAT(N)
      BTL=6.[illegible]*BTL/QZ
```

```
      COMMON/XYA/X(101),Y(101)
      N=101
      DO 15 I=1,10
      X(I)=BTT(I)
      Y(I)=XS(I)
C        CHECK FOR  ARGUMENT  INCONSISTENCY
      IF(M.LT.N)GOTO 2
      TRUBL=1.
      RETURN
C     CALCULATE  FIRST ORDER DIFF
      NM1=N-
      DO 3  I=1,NM1
      TABLE(I,1)=(Y(I+1)-Y(I))/(X(I+1)-X(I))
      IF(M.LE.1)GOTO 6
C       CALCULATE HIGHER-ORDER DIFF
      DO 5  J=2,M
      DO 5  I=J,NM1
      ISUB=I+1-J
      TABLE(I,J)=(TABLE(I,J-1)-TABLE(I-1,J-1))/(X(I+1)-X(ISUB))
      TRUBL=0.0
      RETURN
      END
```

```
$IBFTC FNEW
      FUNCTION  FNEWT(M,IDEG,XARG,TRUBL)
      COMMON /YL/TABLE(101,4)
      COMMON/XL/BTT(101),XS(101)
      COMMON/XYA/X(101),Y(101)
      N=101
      IF(IDEG.LE.M)  GOTO  2
      FNEWT=0.0
      TRUBL=1.
      FNEWT=0.0
      RETURN
         SEARCH  X-VECTOR FOR ELEMENT.GE.XARG
      DO  4  I=1,N
      IF(I.EQ.N.OR.XARG.LE.X(I)) GOTO  5
      MAX=I+IDEG/2
      IF(MAX.GT.N) MAX=N
      IF(MAX.LE.IDEG) MAX=IDEG+1
C      COMPUTE  INTERPOLANT--ALUE
      YEST=TABLE(MAX-1,IDEG)
      IF(IDEG.LE.1) GOTO  13
      IDEGM1=IDEG-
      DO  12  L=1,IDEGM1
      ISUB1=MAX-
      ISUB2=IDEG-1
2     YEST=YEST*(XARG-X(ISUB1))+TABLE(ISUB1-1,ISUB2)
3     ISUB1=MAX-IDEG
      TRUBL=0.
```

```
      FNEWT=Y(ISUBI)*(XARG-X(ISUBI))+Y(ISUBI)
      RETURN
      END
CIBFTC ADVN
      SUBROUTINE ADVN(ZG,ZP,ERR,[illegible],AG)
C ---------------------------------------------------------------------
C           TH  ADVANCE IN THE CONTACT ANGLE
C     THE ROUTINE FINDS THE ADVANCE IN ENGAGEMENT POSITION WHENEVER
C      THERE IS POSITIVE ERROR BET. YHE IN COMING PAIR OF TEETH
C      BY SUCCESSIVE ITERATION PROCESS
C                  DUE TO THE DEFLECTION
C ---------------------------------------------------------------------
      COMMON/LA/E,PHI,DP,G
      REAL  MU,MG
      XM=1.0/DP
      PHI=20.*3.143/
      MG=ZG/ZP
      VMG=1./MG
      CD=(ZG+ZP)*XM/2.
      RG=(CD*MG)/(1.+MG)
      RP=CD/(1.+MG)
      RGO=RG+XM
      RPO=RP+XM
      RGB=RG*COS(PHI)
      RPB=RP*COS(PHI)
      BEA=ARSIN(RG*SIN(1.5715+PHI)/RGO)
      ALPA=1.5715-PHI-BEA
      CE=SQRT(RGO**2+RG**2-2.0*RGO*CD*COS(ALPA))
      BETA=ARSIN(RGO*SIN(ALPA)/CE)
      THETA=0.
20    ETA=ALPA+ERR/RGO+THETA
      DC=SQRT(CD**2+RGO**2-2.0*CD*RGO*COS(ETA))
      XLM1=ARCOS(RRB/[illegible])
      XLM2=ARCOS(RRB/[illegible])
      XINLM1=TAN(XLM1)-XLM1
      XINLM2=TAN(XLM2)-XLM2
      DELT =XINLM1-XINLM2
      MU=ARSIN(RGO*SIN(ETA)/DC)
      THETAG=(MU-BETA-DELT)*VMG
      IF(ABS(THETAG-THETA).LT.0.000001)  GO TO 5
      THETA=THETAG
      GO TO 20
5     RETURN
      END
CIBFTC FNT
      FUNCTION FNT(X,Y,N,XARG)
      DIMENSION  X(1),Y(1)
      DO  4  I=1,N
      IF(I.EQ.N.OR.XARG.LE.X(I)) GOTO  5
5     IF(I.EQ.1) I=2
      FNT=(XARG-X(I))*(Y(I-1)-Y(I))/(X(I-1)-X(I))+Y(I)
```